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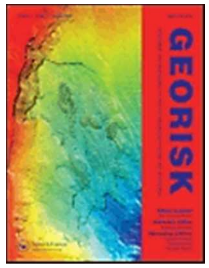

Structural stability of gravity dams: a progressive assessment considering uncertainties in shear strength parameters

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Abstract

Structural sliding stability of gravity dams is most often quantified using a binary safe/unsafe criterion such that the shear resistance has to be larger than the driving shear load. Large deterministic factors of safety, FS_{det} (e.g. 3 in normal condition), are used in existing guidelines to guard against material and loads uncertainties. Some guidelines allow an arbitrary reduction in FS_{det} (e.g. 2) when the knowledge in strength parameters increases from material test data. Yet, those reduced FS_{det} are not based on a rational consideration of uncertainties. Propagation of uncertainties could be done using comprehensive probabilistic analyses, such as Monte-Carlo simulations (MC). However, MC are complex and challenging for practical use. There is thus a need to develop simplified reliability based safety assessment procedures that could rationalise the adjustment of FS_{det} from existing dam safety guidelines. This paper presents a progressive analysis methodology using four existing safety evaluation formats of increasing complexity: (i) deterministic, (ii) semi-probabilistic (partial coefficient), (iii) reliability based *Adjustable Factor of Safety (AFS)*, and (iv) probabilistic (MC). Comprehensive comparisons are made for the sliding safety evaluation of a 80 m gravity dam. Results are presented in terms of sliding factors of safety, allowable water levels, and demand/capacity (load and resistance) ratios. It is shown that the reliability based AFS formulation, using direct integration, is simple and practical to use in complement to existing dam safety guidelines before undertaking MC simulations. AFS yielded results with a maximum difference of approximately 10% as compared to rigorous MC probabilistic analyses.

1. Introduction

Dams are major infrastructures, for which failure is an extremely rare event but with very serious consequences. Gravity dams are particularly sensitive to overtopping, due to the large hydrostatic thrust and uplift pressures that reduce the shear strength capacity that could be mobilised by frictional resistance and cohesion along lift joints or the concrete-rock interface. Dams have a useful service life extending for decades, such that aging of materials and the magnitude of anticipated floods evolve with advances in predictive methodologies and changes in the environment. The structural stability of major dams needs to be re-evaluated every 5-10 years according to Hazard Classification Systems (HCS), most often within the legal framework of a governmental regulatory agency.

Structural stability against sliding should satisfy a binary safe/unsafe limit-state stating that the shear resistance, R, has to be strictly larger than or equal to the driving shear load, L. To guard against uncertainties in R and L, large deterministic factors of safety (FS_{det}) are used. These large FS_{det} may be reduced when new knowledge about the material shear strength parameters is acquired to better quantify the friction coefficient and cohesion. For instance, in CDA (2007), $FS_{det} = 3$ if no material test is available, and $FS_{det} = 2$ if tests are done. A better knowledge of strength parameters thus authorises a reduced safety margin. However, the specified FS_{det} numerical values are rounded numbers from experience and not from a rational approach to quantify reduction in strength parameters uncertainties.

There is thus a need to develop, validate and verify simplified reliability based safety assessment procedures that could rationalise the FS_{det} adjustment by referring to data bases developed from evaluating L and R uncertainties readily available from countless test series and load statistics (Kreuzer and Léger 2013). Simplified methods should be robust, computationally efficient, with a limited number of random variables, and easily interpretable. They then can replace the deterministic L and R parameters. Moreover, reliability based safety assessment will also answer a societal aspect in form of relating the required safety margin to a HCS used by the dam safety regulations of the particular country. Of course, comprehensive reliability based probabilistic analyses could be undertaken to propagate uncertainties in structural stability analyses. However, comprehensive probabilistic analyses (e.g. Monte Carlo (MC) simulations) require extensive time, sophisticated tools, expertise and resources that are most often not available. To adjust the required safety margin as a function of a reduction in shear strength uncertainties, this paper presents a progressive analysis methodology of increasing complexity and accuracy. It consists in applying successively (i) deterministic analyses, (ii) simplified reliability based *Adjustable Factor of Safety (AFS)* analyses, and (iii) full probabilistic analyses (crude MC). The main objective of the paper is to establish the range of AFS modelling parameters for which coherent results could obtained with MC simulations that are used as reference values.

This paper is organised as follows. After the review of literature in section 2, section 3 describes four different safety formats: (i) deterministic, (ii) semi-probabilistic (partial coefficient), (iii) reliability based AFS, and (iv) probabilistic (MC). Comprehensive comparisons of the four safety evaluation formats are done by comparing sliding stability indicators of a 80 m-high gravity dam, studied in a previous ICOLD numerical Benchmark seeking to quantify the sliding probability of failure, p_f, as a function of the upstream reservoir water elevation, H_w (Escuder-Bueno et al., 2011). It is shown that coherent results, as compared to MC, could be obtained while using the AFS method when the required safety margin is being compared to the ratio of resistance and load mean values. The AFS method is thus shown as a simple and practical preliminary substitute to full probabilistic MC analysis that may therefore be avoided within the scope of the gravity dam stability problem studied herein. Two additional AFS safety margin formulations have also been investigated to demonstrated its range of applicability as compared to MC (i) when all uncertainties have been factored in AFS parameters such that AFS ≥ 1, and (ii) when an additional safety margin is deemed necessary to account for ignorance, or lack of information in spite of having considered uncertainties with all AFS factors (Kreuzer and Leger 2013).

2. Stability assessment of gravity dams considering uncertainty analysis

One can solve the gravity dam stability problem as a deterministic one where there is no uncertainty. At the other end of the spectrum, total ignorance of phenomena that may affect stability, and its outcome, cannot be quantified because classification and identification of what is unknown is not possible (Riley, Webley and Thomson 2017; D. Rumsfeld famous deep "*unknown-unknown*" uncertainty; Baecher 2016). We will therefore focus our analysis on epistemic and aleatory uncertainties in input data (friction coefficient and cohesion) that admit statistical descriptions from a frequentist point of view.

Deterministic analysis is traditionally used to assess the stability of dams (ANCOLD 2013; CDA 2007; FERC 2002; Ruggeri 2004; USBR 1976; USACE 1995, 2005). Deterministic methods do not allow accounting explicitly for uncertainties in strength capacity and load effects (stresses). Moreover, deterministic FSs are round numbers without rational safety-relevant significance, apart from grading them to the frequency of load combination (usual, unusual, extreme).There is thus a huge interest in the profession to move towards more refined methods to consider uncertainties.

Probabilistic analysis allows considering nature's randomness, and human incomplete knowledge within the probabilistic safety evaluation. Mathematical constructs are used normally presented by probability density functions and engineering judgment. Then, comparisons between several failure scenarios are possible while assessing probability of failure, p_f, and related risk according to the consequence model. Probabilistic methods are thus very useful tool to decision-making (Bury and Kreuzer 1985; FERC 2014). Probabilistic risk analysis, and associated risk management, are used for other civil structures and are now in active development in dam engineering (Baecher 2016; FERC 2014; Hartford and Baecher 2004; Hartford et al. 2016; Kalinina et al. 2016; Peyras et al. 2010; Peyras et al. 2012; SPANCOLD 2013; Westberg Wilde and Johansson 2016; Zhang et al. 2016). As dam engineering is moving towards models accounting for uncertainties, ICOLD organized an International Benchmark Workshop on Numerical Analysis of Dams in 2011 to examine probabilistic analysis (Escuder-Bueno et al., 2011). A new ICOLD Benchmark on Risk Assessment took place in 2017 (Johansson et al. 2017). Risk assessment uses probabilistic methods, and consists (i) to combine failure modes to determine a failure scenario by using event or failure trees, (ii) to model failure modes, (iii) to assess the failure probability for each failure mode, (iv) to deduce the total dam failure probabilities. The concept of "*As Low As Reasonably Possible (ALARP)*" is then used in several dam safety guidelines to answer the difficult question "*How safe is safe enough?*" (Bowles 2007). ALARP is used to accept or reject the resulting failure probability and related consequences. Recognising that absolute safety cannot be achieved, the mathematical concept of risk is used to specify reasonable efforts to avert losses considering what can be achieved with the available resources. Reasoning in terms of probability of failure and acceptable risks is a key societal construct, subjected to time evolution. While accepting the ALARP principle, it is then possible to make the transition between the deterministic world, where achieving the required FS_{det} is believed to provide absolute safety, and the probabilistic world were uncertainties are explicitly recognised (Luhmann 2005).

A probabilistic analysis could account for uncertainties in cohesion, friction, drain efficiency, and several other parameters as shown in Altarejos-Garcia et al. (2015, 2012). Probabilistic analysis also allows reliability assessment of dam-foundation-reservoir systems (Westberg Wilde and Johansson 2013). In reliability based safety assessment, fragility analysis, first used in seismic evaluation of nuclear power plants, is a key step (Porter 2017). Fragility analysis is the computation of the probability to reach an undesirable limit-state for a known loading intensity. Fragility curves, expressed as function of reservoir elevation, H_w, provide quantitative cumulative distribution functions for the dam to resist sliding. Fragility curves provides a rational tool that could be used to compare several remedial options if a need for strengthening is identified (Ebeling et al. 2012; Ellingwood and Tekie 2001; Tekie 2002).

However, probabilistic assessments require many parameters to describe uncertainties, such as random variables, Probability Density Functions (PDF), PDF bounds, coefficients of variation, which may affect substantially the analysis results as shown by the wide dispersion in fragility curves computed from the participants in the ICOLD Benchmark (Escuder-Bueno et al. 2016). Then, the decision to take remedial action, if necessary, depends on a complex and possibly not unique assessment: one may wonder about limits of reliability methods, which are finally not the unerring reference (Kreuzer 2000, 2003).

Semi-probabilistic (partial coefficient) analysis is a first simplified approach that allows considering uncertainties according to each load and resistance parameter (ANCOLD 1991; CFBR 2013, 2015; IS 1984-1998; Peyras et al. 2008; Rocha 1974; SPANCOLD 2003). Partial safety coefficients are ideally calibrated according to probabilistic analyses. However, each dam is a unique hydro-geomechanical system. Calibration may not be based on consistent sampling, but adapted to correspond to existing structures designed with deterministic methods (Jongejan and Calle 2013; Kovarik 2000). In the Netherlands, the National Flood Risk Analysis project developed recommendations evolving from deterministic to semi-probabilistic analysis to assess stability of hydraulic structures (levees), using probabilistic analysis to calibrate coefficients according to target failure probability, p_f* (Jongejan and Maaskant 2013; Vergouwe 2016).

Adjustable Factor of Safety (AFS) analysis is another simplified and practical way to perform probabilistically (reliability) based safety assessment. AFS considers only two basic random variables, the resistance **R** and the load, **L**. AFS is seeking a binary outcome such that AFS $\geq FS_{req}$, where FS_{req} is depending on a user defined target p_f* or a target reliability index, β*. As described in more details in section 3, AFS connects R and L within a probabilistic framework using their PDF along with six uncertainty factors calibrated on empirical evidences, including coefficients of variations as well as upper (for L) or lower (for R) PDF bounds (Kreuzer and Léger 2013).

A progressive approach, with increasing level of complexity to assess the sliding stability of gravity dams is therefore proposed as follows: (i) deterministic analysis is done first, (ii) then semi-probabilistic method can optionally be used to distinguish uncertainties about the friction coefficient and cohesion. (iii) The AFS method is next performed to rationally adjust the computed FS_{det} and compared it to FS_{req} considering a probabilistic description of R uncertainties and a selected target p_f* or β*. Finally, (iv) full probabilistic analysis may be undertaken if deemed necessary.

3. Specification for required safety margins in investigated safety evaluation formats

3.1 Deterministic

Deterministic analysis is traditionally used to design and assess dam stability. It consists in defining a factor of safety (FS) between the dam's resistance R and the loading L from $FS = R/L$, and to compare it to required values according to applicable guidelines. The gravity method and Mohr-Coulomb criterion are most often employed to define the shear strength resistance to compute sliding FS. Typically, three values for resistance are considered in parametric study: best estimate, lower bound, and upper bound. Different load combinations are considered, only those associated with water levels are studied herein: usual, unusual, and extreme (flood). Moreover, required FS relate to dam-foundation interface and lift joints.

Some guidelines require FS without any consideration to the level of knowledge about strength parameters (Table 1). Some other guidelines recommend FS considering the level of knowledge in strength parameters. The required FS are larger if no material tests have been realised (Table 2).

Table 1. Required deterministic factors of safety without any explicit consideration of uncertainties.

Load combination	**Required factor of safety**		
	UBSR (1976)	USACE (1995)	FERC (2002) [(a)(b)]
Usual	3.0	2.0	3.0
Unusual	2.0	1.7	2.0
Extreme	1.0	1.3	-

[(a)] Dams are categorised according to the consequences of a failure: the coefficients are for a dam with moderate to high risks.
[(b)] Other FS are required for friction only (cohesion is null).

Table 2. Required deterministic factors of safety depending on the level of knowledge of strength parameters.

Load combi-nation	**Required factor of safety**					
	CDA (2007) [(a)]		ANCOLD (2013) [(a)]		USACE (2005) [(b)]	
Knowledge?	*No tests*	*With tests*	*Not well-defined* [(c)]	*Well-defined* [(c)]	*Well-defined* [(d)]	*Ordinary*
Usual	3.0	2.0	3.0	2.0	2.0	1.7
Unusual	2.0	1.5	2.0	1.5	1.5	1.3
Extreme	1.3	1.1	1.5	1.3	1.1	1.1

[(a)] Other FS are required for friction only (CDA 2007) or residual values for C, tanϕ (ANCOLD 2013).
[(b)] Dams are categorised according to the consequences of a failure: the FS are for a dam with moderate to high risks.
[(c)] According to ANCOLD (2013), "well-defined" means that "a sufficient number of tests have been done to specify the strength parameters with reasonable certainty (e.g. assumed strength is exceeded by 80% of the test results from a test regime involving a significant number of tests)".
[(d)] According to USACE (2005), site information is "well-defined" when records are available, dam is monitored, uplift are known, and "foundation strengths can be established with a high level of confidence".

3.2 Semi-probabilistic (partial coefficients)

Rocha (1974) suggested to introduce partial safety coefficients for friction and cohesion instead of a single FS. This approach is to account for different levels of uncertainties in these two shear strength mechanisms. Divisor coefficients of 1.5 to 2, and 3 to 5, could be applied respectively to the friction coefficient, tanϕ, and the cohesion, C. It is thus recognised that uncertainties in cohesion are more important than in friction, whereas there is no mathematical justification for the recommended values. Thereafter, countries like India (IS 1984-1998) and Spain (SPANCOLD 2003) chose to use the concept of partial strength reduction coefficients (Table 3).

Semi-probabilistic assessment consists in using some partial safety coefficients for both loads and strengths, increasing loads and reducing strengths depending on the target reliability of the structural component. These coefficients are ideally calibrated from probabilistic methods, and adapted to existing structures designed with deterministic analysis (Jongejan and Calle 2013; Kovarik 2000).

Table 3. Semi-probabilistic partial safety coefficients

Load combination	Partial safety coefficients							
	IS (1984-1998)		SPANCOLD (2003) [b]		CFBR (2013)		ANCOLD (1991) [c]	
	γ_C [a]	$\gamma_{\tan\phi}$	γ_C	$\gamma_{\tan\phi}$	γ_C	$\gamma_{\tan\phi}$	Strength	Load [d]
Usual	3.6	1.5	5.0	1.5	3.0	1.5	0.3	
Unusual	3.6	1.5	4.0	1.2	2.0	1.2	0.4	
Extreme – flood	1.2	1.0	3.0	> 1.0	1.0	1.0	0.8	

[a] C is the cohesion, $\tan\phi$ is the friction coefficient; γ_C and $\gamma_{\tan\phi}$ are associated partial safety coefficients such that semi-probabilistic sliding stability limit-state criterion is: $\frac{A_c.C/\gamma_C + V.\tan\phi/\gamma_{\tan\phi}}{L} > 1.0$ where V is the sum of vertical forces, A_c the compressed sliding area, L the hydrostatic thrust (IS 1984-1998; SPANCOLD 2003; CFBR 2013).
[b] Dams are categorised according to the consequences of failure: the coefficients are for a dam with moderate to high risks.
[c] Multiplier coefficients applied to strength and load parameters to compute R′ and L′ such that semi-probabilistic sliding stability limit-state criterion is: $R' > L'$ (ANCOLD 1991).
[d] Coefficients for loads are 0.95 for water and well-known dead loads contributing to stability, 0.90 for not well-known concrete weight, 1.05 for water, uplift and dead loads contributing to instability, 1.50 for live and silt loads contributing to instability.

3.3 Probabilistic

Deterministic FS and partial safety coefficients do not inform about the safety margin including uncertainties about parameters employed in analysis. Probabilistic analysis allows to compute a failure probability, p_f, considering mathematically uncertainties. HCS have been developed to differentiate structures according to the consequences a failure.

The ALARP concept has often been used to link loss of human lives (or persons in danger), more than material costs of consequences, to recommended acceptable failure probability (Figure 1).

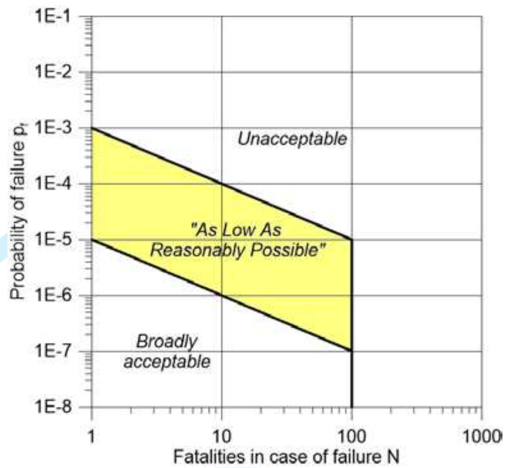

Figure 1. Concept of "As Low As Reasonably Possible" adapted from (CDA 2007).

In probabilistic analysis, load (L) and resistance (R) parameters are statistically distributed according to PDF selected to be representative of tests and knowledge about these parameters. Failure develops when internal load demand exceeds the resistance capacity of the dam. Influences of different selections of PDF data in probabilistic analyses have been studied in Altarejos-Garcia et al. (2012); Carjaval et al. (2009a, 2009b, 2009c); Carjaval, Peyras, and Baconnet (2010); Krounis and Johansson (2012); Krounis et al. (2016); Lombardi (1988, 1993, 2006); Spross, Johansson and Larsson (2013). Probabilistic analysis, no matter how sophisticated, can still lead to very different solutions for a given problem because of the complex choices of random variables, characteristic values, PDF, bounds, which can largely influence final results. For example, Figure 2 illustrates the wide dispersion obtained from participants in an ICOLD Benchmark seeking to compute the fragility curve (Fcurve) for p_f as a function of H_w for a 80 m-high gravity dam given fifteen sets of "cohesion, friction angle" data pair representing material test data. The detailed description of the problem is given in section 4.1 of this paper. In Figure 2, we present results from all participants that used MC simulations, (Fcurves 1-7) as well as our own MC solutions (Fcurves 8-12). Using the same PDF data for C and tanϕ, ICOLD Fcurve 2 is similar to Fcurve 8 computed herein for water levels ranging from 75 m to 80 m. Our MC probabilistic analysis procedure is thus validated for the selected PDF data.

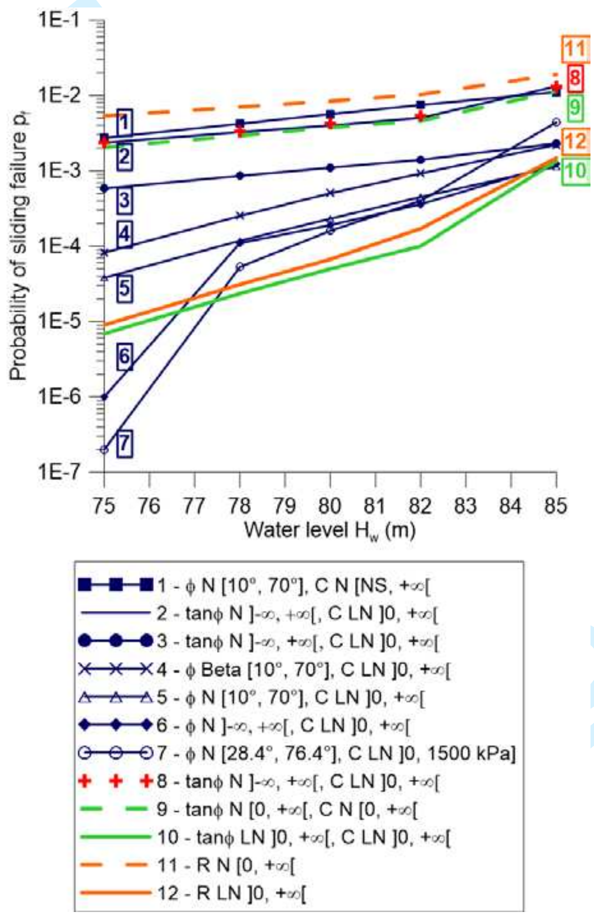

Figure 2. Dispersion in fragility curves computed for the ICOLD Benchmark for a 80 m-high gravity dam (Fig. 5); ϕ is the friction angle, C the cohesion, R the global resistance of the dam defined in Eq. 2; N = normal distribution, LN = lognormal distribution; selected bounds are indicated. Fcurves1 to 7 are from participants in the ICOLD Benchmark, Fcurves 8 to 12 are our solutions to the Benchmark.

3.4 Reliability based Adjustable Factor~~s~~ of Safety: AFS

Kreuzer and Léger (2013) presented a simplified reliability based method to assess dam stability. It depends on two uncertain random variables, R and L. An *Adjustable Factor of Safety (AFS)* is defined by:

$$AFS = \frac{\mu_R\{1-(k_R.c_R+\alpha_R)\}}{\mu_L\{1+(k_L.c_L+\alpha_L)\}} = FS_{det}.U_{RL} \qquad \text{(Eq. 1)}$$

FS_{det} is the deterministic factor of safety, c_R and c_L are the coefficients of variation related to physical uncertainties, the natural intrinsic dispersion of values, k_R and k_L are related to statistical uncertainties defined herein as the lack of knowledge, from the number and reliability of material test data and α_R and α_L are related to model or (epistemic) uncertainties (Figure 3). Comprehensive description and numerical values for these coefficients have been suggested in Kreuzer and Léger (2013) depending on the knowledge of the structure.

Considering only two random variables, R and L, the AFS aims to be compared to a Required Safety Factor, FS_{req}, depending on a target failure probability, p_f*, or the corresponding reliability index β*. For a safe structure, the stability criterion becomes $AFS \geq FS_{req}$. FS_{req} is computed iteratively by direct integration to be the $FS = \mu_R/\mu_L$ such that with the selected PDF data for R and L, the computed p_f would correspond to the target failure probability, p_f*. PDF are bounded at distances from the mean corresponding to a number k_R or k_L of standard deviations, on the left for the resistance and on the right for the load. For unbounded PDF, tails of distributions are considered in the computation of FS_{req} (it would correspond to $k = \infty$), but a k value has to be defined for the computation of AFS. FS_{req} decreases when the uncertainties in L and R are reduced. The AFS is a simple approach to introduce uncertainties using PDF data of the basic random variables **L** and **R** directly into the safety evaluation process. It allows to study the effect of reducing the coefficient of variation of the resistance, $c_{\mathbf{R}}$, using material tests, on a rational basis as opposed to existing deterministic dam safety guidelines using arbitrary reduced FS requirements. The AFS considers only p_f*, that could be specified directly from a HCS, avoiding to rely on assessing speculative terms of risk which require a complementary loss model. A user-friendly open source computer program, **R-AFS**, was developed to perform AFS and FS_{req} computations (Morin 2016). The R-AFS implementation is controlled by an input-output environment, using the "**R**" open-source statistical computational platform (see https://www.r-project.org/). A copy of R-AFS could be obtained by contacting the second author (pierre.leger@polymtl.ca).

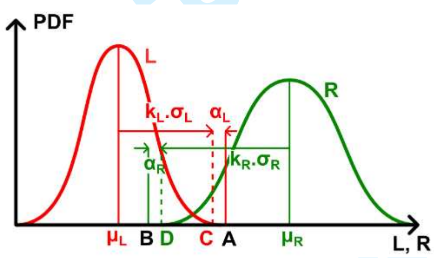

Figure 3. Definition of the six uncertainty coefficients in the reliability-based AFS format: (i) $c_R = \sigma_R/\mu_R$, (ii) k_R, (iii) α_R, (iv) $c_L = \sigma_L/\mu_L$, (v) k_L, (vi) α_L.

The advantages of the reliability-based AFS are (i) the rationality to account for uncertainties using selected PDF data and a target p_f* (or β*) in similarity to probabilistic analysis, (ii) its simplicity and practical use, (iii) a clear interpretation in the form of a binary decision to accept/reject the computed FS.

Of course, if one has the certitude to have properly factored all uncertainties with the c, k and α values, $FS_{req} = 1$ would be adequate. The general accepted safety performance criterion then becomes AFS $\geq$ 1. Another approach is to consider all uncertainties in the computation of FS_{req}. In the gravity dam shear strength problem, we are then seeking to satisfy $(\mu_R/\mu_L) \geq FS_{req}$ $(c_R, k_R, \alpha_R, p_f^*)$. However, an additional safety margin might be provided for initial imperfection, ignorance or lack of information leading to the acceptance criterion AFS $\geq FS_{req}$ $(c_R, k_R, \alpha_R, p_f^*)$ (Kreuzer and Léger 2013). These three acceptance criteria (i) AFS $\geq$ 1, (ii) $(\mu_R/\mu_L) \geq FS_{req}$, and (iii) AFS $\geq FS_{req}$ are compared with MC analyses, used as the reference solution to evaluate the reliability based AFS method.

3.5 Progressive approach to introduce uncertainties

The above safety evaluation formats, ranging from deterministic to comprehensive probabilistic analyses, show different ways to account for uncertainties, from various sources appearing at each level of the stability assessment of the dam: material testing, selection of strength and load parameters, structural model. A progressive approach may then be developed to best account for these uncertainties, from simple to more complex but more precise evaluation formats (Figure 4).

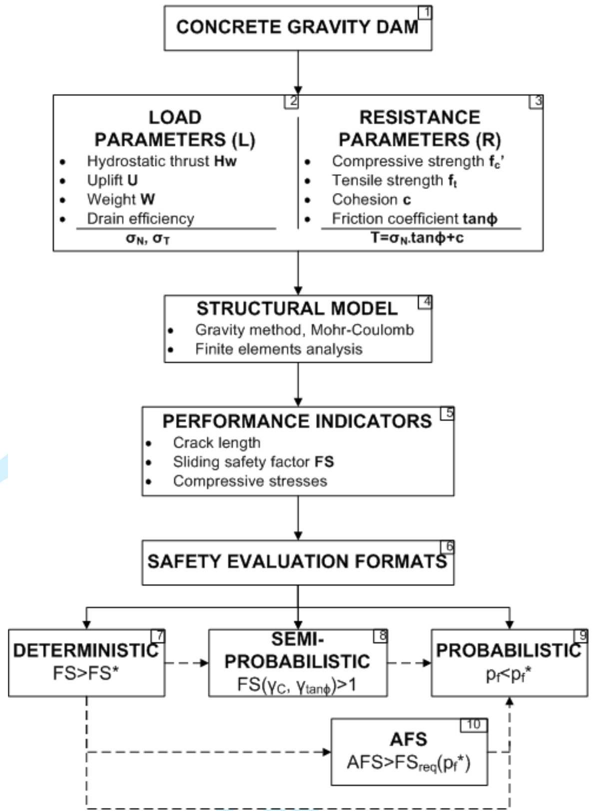

Figure 4. Progressive approach for dam safety assessment: parameters, sources of uncertainties and performance indicators.

4. Application of progressive safety assessment

4.1 Description of the gravity dam for applications

The dam for applications is a 80 m-high concrete gravity dam that might be subjected to overtopping. The dam geometry (Figure 5) is given in the 11th ICOLD Numerical Benchmark (Escuder-Bueno et al., 2011). A single and simple failure mode, corresponding to horizontal sliding along the dam-foundation interface, is to be investigated. In reality, the kinematic of a dam base sliding-turning failure mode might occur along inclined planes propagating in the foundation (Fishman 2009). However, our study is restricted to the ICOLD benchmark problem to allow comparisons with previously published results (Fig. 2). The resistance, **R**, is a function of two basic random variables, (i) the friction coefficient, tanϕ, and (ii) the cohesion, C. The dam weight, W, and the drain effectiveness, E, are considered as given constant parameters. The uplift pressure, U, is a function of the water level, H_w. In this application, there is no uncertainty for the load, **L**. The water level, H_w, is increased systematically to reach an unacceptable limit state. In the case of overtopping, the water weight on the crest is estimated as W_w. The gravitational acceleration, the dam-foundation interface tensile strength, the water and concrete densities used in computations are respectively, $g = 9.81$ m/s², $f_t = 0$, $\rho_w = 1000$ kg/m^3, and $\rho_c = 2400$ kg/m^3. The classical gravity method is used in stability analyses, considering cracking at the dam-foundation interface. If the base crack extends beyond the drain, the full uplift pressure is considered in the crack.

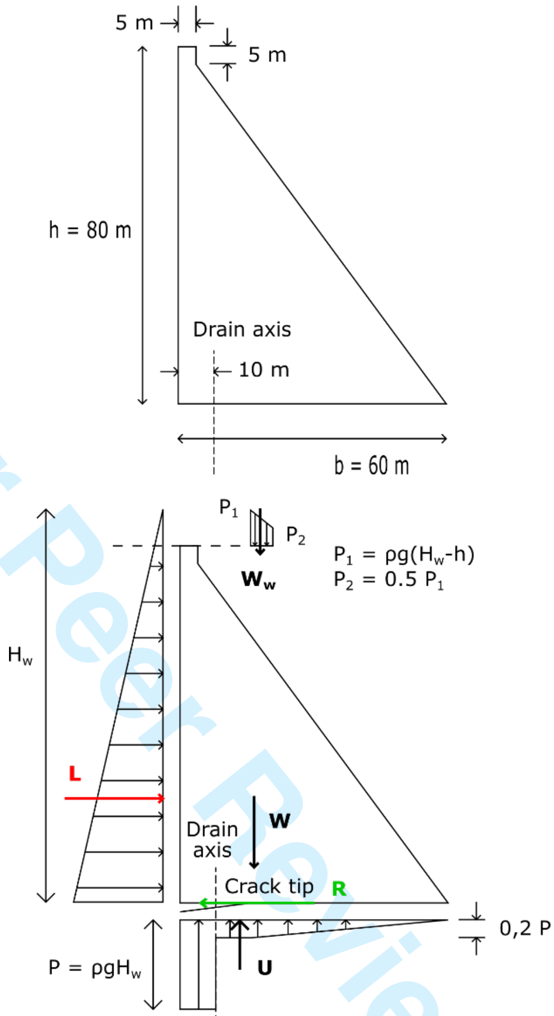

Figure 5. Geometry, load, L, drainage, and resistance, R, (friction and cohesion) properties of the gravity dam analysed.

Fifteen couples of friction angle ϕ (°) and cohesion C (kPa) are specified as input "material test" data in the 11th ICOLD Numerical Benchmark seeking to estimate the sliding probability of failure, p_f, of the dam (see Appendix 1). The statistics for C and the friction coefficient, tanϕ, are summarised in Table 4. The coefficient of variation for cohesion, c_C, is 0.67, which is quite large. Distribution fitting has been realised with N-PDF and LN-PDF. The LN-PDF was found to suit the data best taking into account the skewness, whereas N-PDF is symmetrical.

Table 4. Material test data statistics for friction and cohesion at the dam-foundation interface.

Material test	Cohesion C (kPa)	Friction angle ϕ (°)	Friction coefficient tanϕ
Mean μ – best estimate	367	52.4	1.36
Standard deviation σ	247	7.99	0.39
Coefficient of variation c	0.67	0.15	0.29
Minimum – lower bound	0	37	0.75
Maximum – upper bound	800	63	1.96
5% fractile – N	0	39.3	0.72
5% fractile – LN	112	40.3	0.82

4.2 Deterministic stability evaluation

Deterministic analyses are first realised with mean values selected as best estimates for cohesion, C, and friction coefficient, tanϕ. Smallest and largest values are taken as lower and higher bounds. For the usual and unusual load combinations, the maximal allowable water level, H_w, is computed according to CDA (2007, Table 2), without and with material tests (Figure 6).

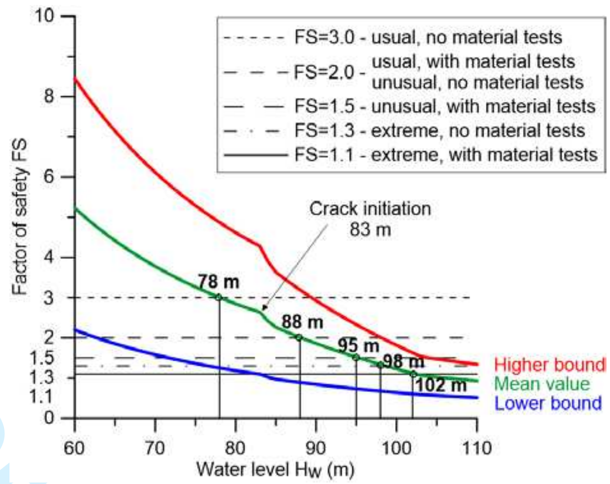

Figure 6. Maximum allowable water level according to deterministic CDA (2007) dam safety guidelines.

These results indicated the importance of having a good knowledge of shear strength parameters. For the usual load combination, the allowable water level increased by 10 m if material tests are realised. However, CDA (2007) does not provide clear guidance on the number of tests, the sampling location and the testing method to be used to obtain representative results with a quantified confidence level. ANCOLD (2013) suggests criteria for the "well-defined" material shear strength parameters (Table 2).

4.3 Probabilistic safety evaluation using Monte-Carlo simulations

A probabilistic assessment requires to select a target p_f*, random variables, their PDF, and their bounds if they are bounded. PDF are bounded at distances from the mean corresponding to a number *m* of standard deviations on the left and on the right. Unbounded PDF corresponds to $m = \infty$. Herein, the p_f* is 10^{-5}, consistent with the ALARP principles for a high-risk dam (CDA 2007). We work with two sets of random variables either (C, tanϕ) or R. The resistance, R, is described as the sum a friction and a cohesion component:

$$R = V.tan\phi + A_c.C \qquad \text{(Eq. 2)}$$

a mean value is computed for R and the standard deviation, σ_R, is estimated from:

$$\sigma_R = \sqrt{\left(V.\sigma_{tan\phi}\right)^2 + (A_c.\sigma_C)^2} \qquad \text{(Eq. 3)}$$

where V is the sum of vertical forces, and A_c the area in compression.

The related PDF are successively selected as N and LN in sensitivity analyses. Bounded and unbounded PDF are also studied. MC computations are realised with MATLAB® (The MathWorks 2016), $n = 10^7$ samples are found adequate to obtain convergence for p_f. For instance, when $p_f = 10^{-5}$, the accuracy is $p_f = 10^{-5} \pm 5.10^{-7}$.

Unbounded PDF are first studied. When the unbounded hypothesis is considered with N-PDF, negative values of C, tanϕ, or R, are replaced by new draws in MC simulations. The results are presented in Figure 7. LN-PDF yielded failure probabilities smaller than N-PDF. The reduction from two random variables, (C, tanϕ), to one random variable, R, gave similar H_w results.

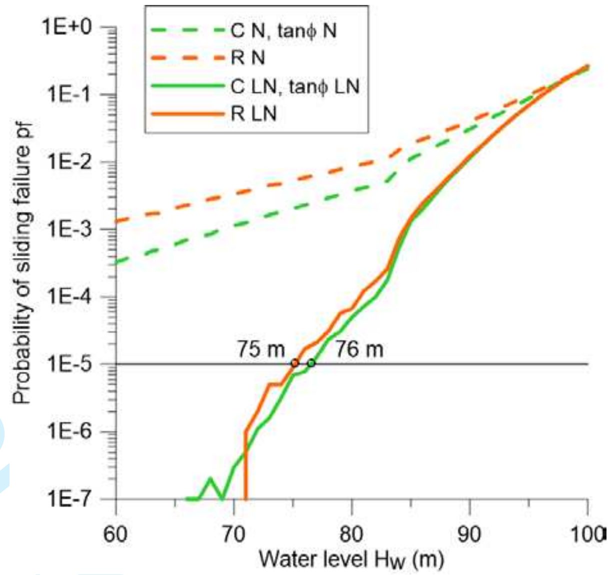

Figure 7. Fragility curves comparing **unbounded** N-PDF and LN-PDF, variables (C, tanϕ) or R, computed with MC; and H_w according to probabilistic analysis for a target failure probability $p_f{}^* = 10^{-5}$.

The effect of ***bounded PDF*** is investigated by selecting values between the 5% fractile for strength parameters on the left of the distribution: m_l standard deviations, and the 95% fractile on the right: m_r standard deviations (Figure 8). For variables C, tanϕ, R, m_l are respectively equal to 1.03, 1.38, 1.39 and m_r respectively equal to 1.88, 1.84, 1.83.

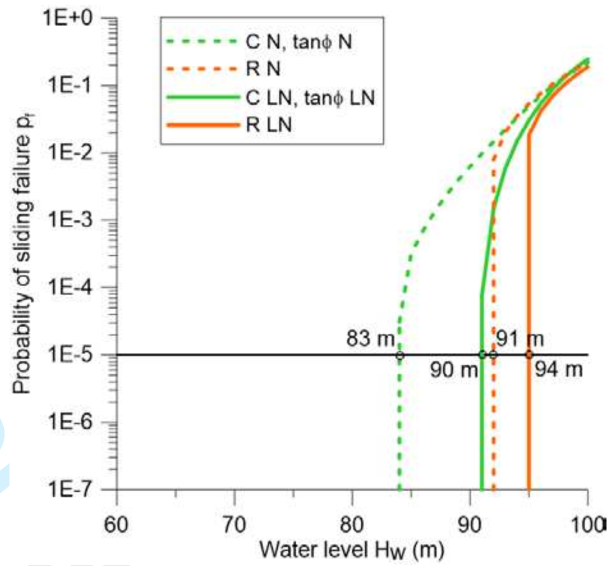

Figure 8. Fragility curves comparing **bounded** N-PDF and LN-PDF at the 5% and 95% fractile values, variables (C, tanϕ) or R, computed with MC; and maximum allowable water level according to probabilistic analysis for a $p_f^* = 10^{-5}$.

Results were similar using N-PDF or LN-PDF, but more sensitive to the selection of random variables. Using a single random variable, R, instead of two (C, tanϕ), yielded higher H_w.

4.4 Semi-probabilistic (partial coefficient) stability evaluation

For the semi-probabilistic analysis, CFBR (2013) suggests as characteristic values, a "wise estimation of the mean", and the 5% fractile if statistical methods are used. Herein, two pairs of (C, tanϕ) are used (i) mean values as best estimates (367, 1.36), and (ii) 5% fractile obtained from the 15 material test data assuming a N-PDF (0, 0.72, Table 4) considered in typical user of CFBR. The partial strength safety coefficients are applied for the usual, unusual, and extreme load combinations (Table 3).

With the extreme combination, the computed H_w was 106 m using the mean values for (C, tanϕ), but as complete base cracking occurred at **104 m** we used this last value as the maximum H_w. Allowable H_w according to each load combination are presented in Figure 9.

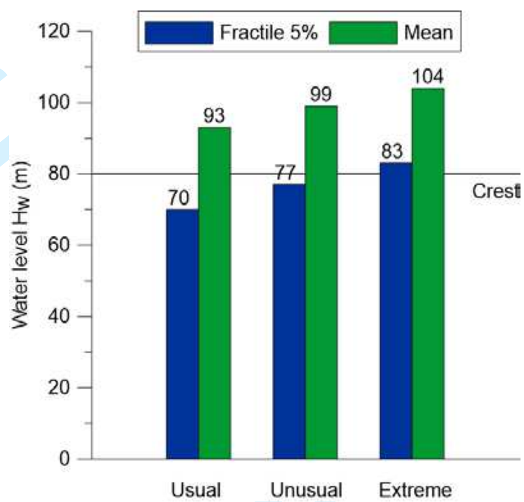

Figure 9. Maximum allowable water level according to semi-probabilistic CFBR (2013) dam safety guidelines.

These results showed a very significant sensitivity of the semi-probabilistic method to the selected characteristic values.

4.5 Reliability based Adjustable Safety Factors – AFS

The selected p_f* is 10^{-5} as recommended for a high-risk dam with good quality assurance and management (Kreuzer and Léger 2013). The AFS method is employed without uncertainties in load L, $c_L = 0$, also, coefficients reporting model uncertainties α_R and α_L

are null. The shear strength random variable in the AFS method is R. The related PDF is LN as recommended in Kreuzer and Léger (2013).

Unbounded PDF are first studied. Tails of distributions are considered while computing FS_{req} but a value for k_R has to be defined for the evaluation of AFS. Values selected for k_R in the AFS computation (Eq. 1) are (i) $k_R = 1.39$, corresponding to the 5% fractile for R for LN-PDF (Holický 2009), (ii) $k_R = 2$, (iii) $k_R = 3$. FS_{req} is computed with unbounded PDF. Computed AFS and FS_{req} are presented in Figure 10. For $k_R = 3$, allowable water level was less than 40 m.

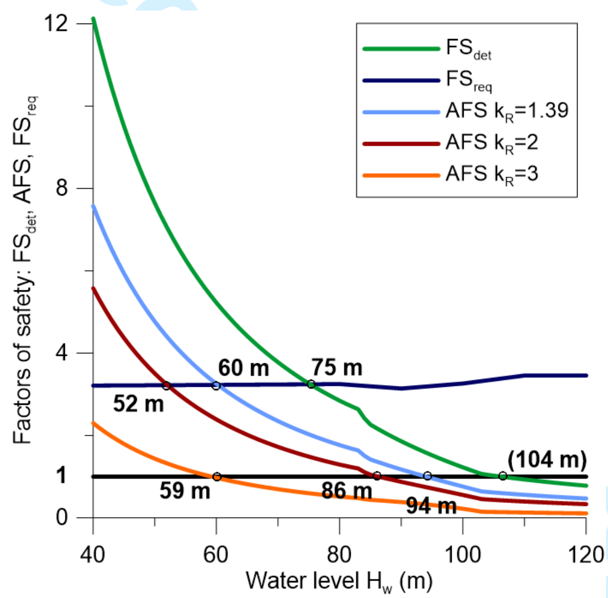

Figure 10. AFS method: deterministic FS_{det}; required FS_{req} for $p_f^* = 10^{-5}$ and **unbounded** LN-PDF for R; and AFS for (i) $k_R = 1.39$, (ii) $k_R = 2$, (iii) $k_R = 3$.

For **bounded PDF** the effect of bounds is investigated by selecting (i) $k_R = 1.39$ (corresponding to the 5% fractile for R for our LN-PDF (Holický 2009), (ii) $k_R = 2$, and (iii) $k_R = 3$ for the computation of AFS as well as FS_{req}. The computed AFS and FS_{req} are presented in Figure 11. For $k_R = 3$, H_w was again less than 40 m.

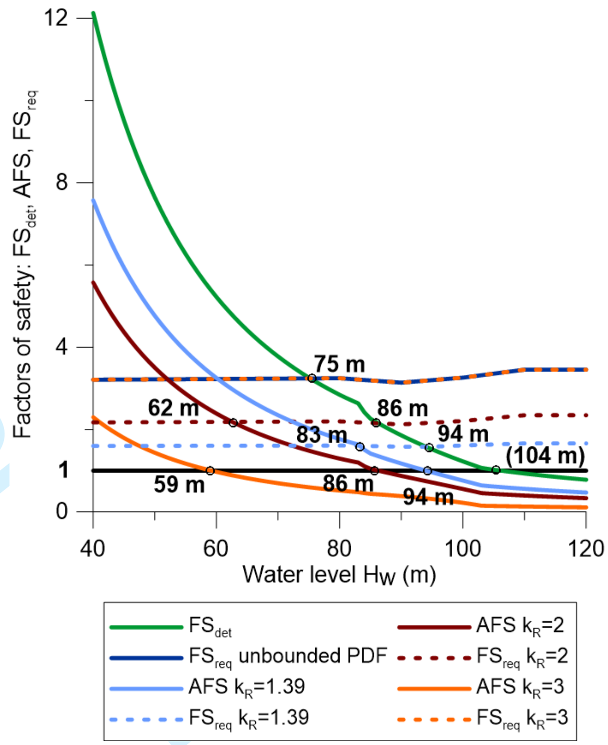

Figure 11. AFS method: deterministic safety factor FS_{det}; required safety factor FS_{req} for $p_f^* = 10^{-5}$ and **bounded** LN-PDF for R; AFS and FS_{req} for (i) $k_R = 1.39$, (ii) $k_R = 2$, (iii) $k_R = 3$.

5. Discussion

5.1 Results from different safety evaluation formats

The 80 m dam was analysed according to four safety evaluation formats applying the proposed progressive safety assessment methodology. The key results are presented (i) in Figure 6 for deterministic analyses, (ii) in Figure 9 for semi-probabilistic analyses, (iii) in Figures 10-11 for AFS, and (iv) in Figures 7-8 for probabilistic MC analyses. Comparative H_w results are presented in Figure 12. Table 5 presents the computed FS for each safety format that are compared to the required FS to declare a safe dam. The reference value to make comparisons and orient the discussion is H_w = 90 m. This allowable H_w is computed from MC simulations, using C and tanϕ as random variables, with LN-bounded PDF

(m_l = 1.03 for C and 1.38 for tanϕ, corresponding to the 5% fractile). It is a reasonable and defensible probabilistic model having considered strength uncertainty in a rational way with two random variables, as well as existing dam safety guidelines to select PDF bounds. Obviously, other reference value for H_w could be selected. However, we present coherent hypotheses moving from one level of complexity to the next such that meaningful comparisons and discussion could be established.

The ***deterministic*** format criteria (Table 2, CDA 2007) are unable to consider the large coefficient of variation in shear strength parameters, H_w being **102 m** for extreme conditions (flood) if material tests had been realised. A parametric analysis showed that lower bound of shear strength data would authorise H_w equal to **82 m**.

Probabilistic sensitivity analyses were applied with random variables (C, tanϕ) or R, LN-PDF, and unbounded or bounded distributions, with $p_f^* = 10^{-5}$. For the unbounded case, H_w was **76 m** with variables (C, tanϕ) and **75 m** with variable R, leading to similar results. With bounds corresponding to the 5% fractiles for strength parameters, H_w was **90 m** (the reference value) for variables (C, tanϕ) and **94 m** for R. Probabilistic analysis (MC) may be considered as the most rigorous approach but is shown to be sensitive to the selection of random variables and PDF bounds.

In ***semi-probabilistic analysis,*** two pairs of values for (C, tanϕ) were used. For the extreme combination, using the mean, H_w was **104 m.** Using the 5% fractile, H_w was **83 m**. This 83 m value was the same as using bounded N-PDF with variables (C, tanϕ) in probabilistic analysis (Figure 8), meaning that calibration of partial coefficients in semi-probabilistic analysis appears to be consistent with results of N-PDF bounded probabilistic analysis.

The ***reliability-based adjustable safety factors (AFS)*** with criterion "AFS $\geq FS_{req}$" yielded very low H_w for unbounded LN-PDF. For unbounded PDF, H_w values were very sensitive to the coefficient k_R. It is deemed inadequate in our application.

The criterion "AFS $\geq$ 1" gave the same H_w for unbounded and bounded LN-PDF because this criterion is not related to the computation of FS_{req}, using PDF data. For $k_R = 1.39$ and $k_R = 2$, H_w obtained with the criterion "AFS $\geq$ 1" are the same as those obtained with criterion "$(\mu_R/\mu_L) \geq FS_{req}$". The criterion "$(\mu_R/\mu_L) \geq FS_{req}$" gave the same H_w for unbounded PDF even for $k_R = 3$. This means that bounding PDF with large k_R is equivalent in ~~RB~~AFS to consider the whole content of the distribution.

5.2 Comparisons of different safety evaluation formats

For the same 80 m gravity dam stability problem, with known 15 pairs (C, tanϕ) and considering (i) no uncertainty, (ii) uncertainties believed to be known with certainty either in RBSF or probabilistic analysis or, (iii) uncertainties with an added safety margin, may decrease the allowable H_w from 104 m to 60 m (Figure 12). Table 5 presents FS computed by each safety evaluation format and the associated safety criterion for two water levels: 80 m and 90 m. Demand/capacity ratios (D/C) have been computed in each case as $(H_w^2/90^2)$ because the applied hydrostatic thrust, L, could be estimated as $L= (\rho_w\, g\, H_w^2)/2$. D/C ratios are presented in enclosed boxes in Figure 12, and for varying values of PDF bounds, m_l (MC) and k_R (AFS), in Figure 13.

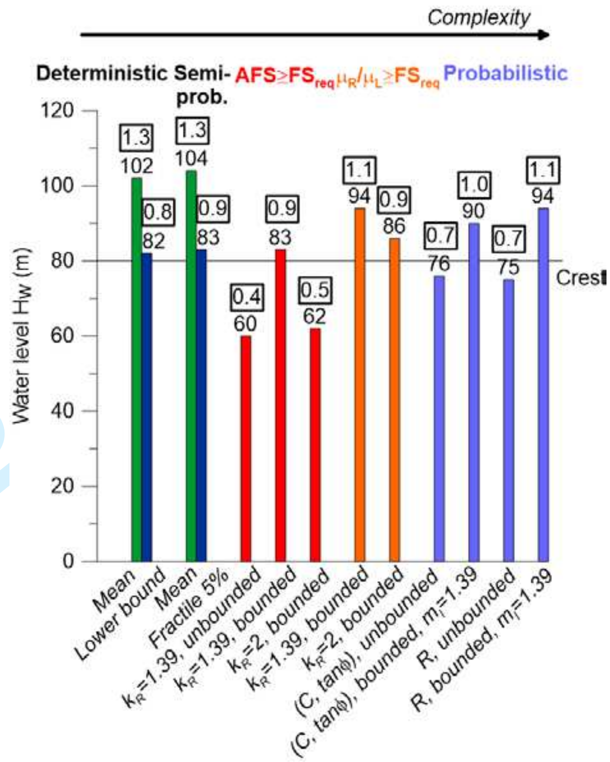

Figure 12. Maximum allowable water level for different safety formats: (i) deterministic, (ii) semi-probabilistic, (iii) AFS criterion "AFS ≥ FS_{req}", (iv) AFS criterion "(μ_R/μ_L) ≥ FSreq", (v) probabilistic MC simulations with random variables (C, tanϕ), (vi) probabilistic MC simulations with random variable R. LN-PDF is used for AFS and probabilistic methods. In boxes are Demand/Capacity ratios.

Table 5. Sliding FS from safety formats: (i) deterministic (CDA 2007), (ii) semi-probabilistic (CFBR 2013), (iii) three criteria of AFS method, (iv) probabilistic (MC simulations).

	80 m - unusual		**90 m - extreme**	
Deterministic	FS_{det}	FS_{det}^{req}	FS_{det}	FS_{det}^{req}
FSdet $\geq FS_{det}^{req}$	2.84	≥ 1.5	1.86	≥ 1.1
Semi-prob.	FS'_{unus}	FS'^{req}	FS'_{extr}	FS'^{req}
$FS' \geq FS'^{req}$	2.14	≥ 1.0	1.86	≥ 1.0
ASF [(a)]	AFS	AFS^{req}	AFS	AFS^{req}
(i) AFS ≥ 1	1.77	≥ 1.0	1.17	≥ 1.0
(ii)	μ_R/μ_L	FS_{req}	μ_R/μ_L	FS_{req}
$(\mu R/\mu L) \geq FS^{req}$	2.84	≥ 1.61	1.86	≥ 1.58
(iii)	AFS	FS_{req}	AFS	FS_{req}
AFS $\geq FS^{req}$	1.77	≥ 1.61	1.17	< 1.58
Probabilistic [(b)]	FS_{pr}	FS_{pr}^{req}	FS_{pr}	FS_{pr}^{req}
$FS_{pr} \geq FS_{pr}^{req}$	1.26	≥ 1.0	1.0	≥ 1.0

[(a)] AFS computations with bounded PDF and $k_R = 1.39$.
[(b)] Reference value from probabilistic analyses is 90 m. FS_{pr} is defined by the inverse of the demand/capacity ratio.

Deterministic and semi-probabilistic formats do not allow to account for uncertainties in parameters used for computations. The allowable H_w were especially high using mean values as strength parameters (102 m and 104 m an allowable capacity approximately 30% larger than the reference value). Using lower bound or 5% fractile values as strength parameters in a sensitivity analysis yielded much smaller H_w (82 m and 83 m), because of the large scatter in test data.

Application of AFS method (computation of FS_{req} and AFS) allows to quantify uncertainties in the safety evaluation. Comparing (μ_R/μ_L) to FS_{req} is then a mathematically expressible safe/unsafe criterion. Allowable H_w were 94 m and 86 m for bounded PDF with k_R respectively equal to 1.39 and 2. This criterion yielded the same H_w as probabilistic MC simulations using variable R and bounded at 5% fractile. This is because direct integration used in computation of FS_{req} and MC simulations give the same p_f. While using $(\mu_R/\mu_L) \geq FS_{req}$, H_w is 94 m corresponding to a D/C of 1.1 using the 90 m reference value. A maximum difference of 10%, as compared to the reference solution, is deemed acceptable for a simplified method. This difference decreases while increasing the PDF bound such that AFS is found to have the same range of applicability as that of MC with a maximum difference of the order of 10%. The criterion "AFS $\geq$ 1" gave also 94 m and 86 m for bounded PDF and k_R equal to 1.39 and 2, respectively. The range of applicability of the criterion "AFS $\geq$ 1" is indicated in terms of D/C ratios in Figure 13. If we accept a difference of 10% with the reference solution, the use of "AFS $\geq$ 1" is restricted to k_R value smaller than 2.5. On the other hand, comparing AFS to FS_{req} allows to introduce an additional safety margin. This added safety margin obviously yields to significantly lower allowable H_w and smaller D/C ratios as compared to other safety formats.

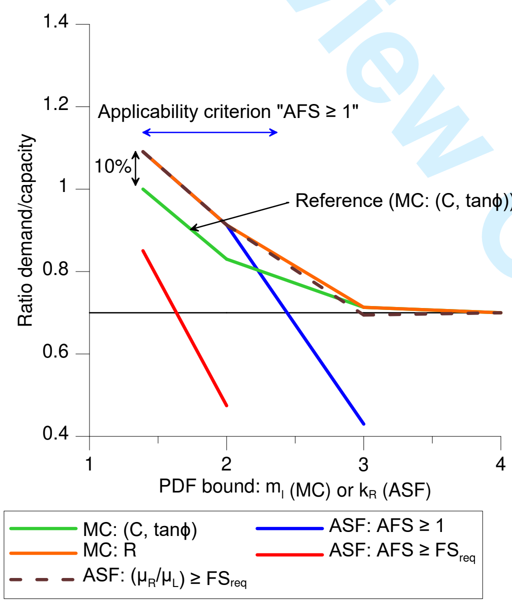

Figure 13. Demand/Capacity ratios for AFS as compared to MC, for varying PDF bounds in MC simulations (m_l) and AFS (k_R)

From a practical standpoint, we feel confident to allow H_w equal to 83 m (3 m of overtopping) for the ICOLD Benchmark dam, using the "$AFS \geq FS_{req}$" criterion with $k_R = 1.39$ corresponding to the 5% fractile of the shear strength parameters. After performing the "sophisticated" analyses presented in this paper, we checked our findings from McCann et al. (1985) against rules of thumb for the allowable depth of overtopping, h_o, as a function of the dam height, using h = 80 m (262.4 ft), in our case. Preliminary screening values for h_o are based on field experience and engineering judgement. They were recommended using the following description and equations (in feet) (i) for dams in good condition: with very little seepage, no cracks or movement ($h_o = h^{0.6} \approx 8.6$ m), (ii) for dams in a fair condition: with moderate seepage, small structural cracks, slight differential movement ($h_o = h^{0.45} - 1 \approx 3.5$m) and (iii) for dams in poor conditions: with excessive seepage, large continuous cracks, excessive differential movements ($h_o = h^{0.3} - 1 \approx 1$ m). The proposed "$AFS \geq FS_{req}$" criterion yielded a h_o value of 3 m corresponding to the rule of thumb for a dam in fair condition. This sounds about right considering the potential scour at the downstream toe, the vibrations induced by the overflowing aerated water nappe, the increased in downstream toe uplift pressure due to the downstream face water jet changing direction at the toe. Such phenomenon are not being considered explicitly in dam safety guidelines, and are very difficult to model from a sound probabilistic standpoint.

6. Conclusions

In this paper, the consideration of material uncertainties in gravity dam sliding stability assessment was investigated for four safety evaluation formats: (i) deterministic, (ii) semi-probabilistic (partial coefficient), (iii) reliability-based Adjustable Factor of Safety (AFS), and (iv) probabilistic Monte Carlo (MC) analysis. The results were presented in terms of the allowable water level, H_w, demand/capacity ratios (D/C), and FS to reach an unstable condition. In AFS and MC, the selected target failure probability, p_f*, was 10^{-5}. A 80 m-high gravity dam was used for applications without considering uncertainties in the applied loads, L. The main conclusions can be summarised as follows:

- The dam engineering profession shows a huge interest in comprehensive probabilistic methods. However, computation of p_f is very sensitive to the selection of shear strength random variables and PDF data as shown by the wide dispersion observed from the ICOLD Benchmark's results for a 80 m-high gravity dam. Practical applications are thus challenging, and generalisation of probabilistic analyses needs clear guidance.
- Using the deterministic format, H_w was found to be 103 m as compared to a reference MC solution with H_w equal to 90 m. The Deterministic safety format was found inadequate to introduce uncertainties even with an arbitrary reduction of the required FS if material tests are conducted
- In MC analyses, PDF bound data are the predominant parameters affecting the computation of p_f. A simplified bounded MC solution, using a single force resultant shear strength random variable, R, yielded a dam capacity approximately 10% larger than a bounded reference MC solution using two random variables (cohesion, C and friction, tanϕ). This excessive capacity decreases as the PDF bound becomes larger, the difference becoming insignificant when unbounded distributions are considered.
- A reliability-based adjustable safety factor (AFS) is a simplified and practical approach to introduce probabilistic uncertainties in shear strength parameters. The criterion

"$(\mu_R/\mu_L) \geq FS_{req}$", using direct integration to compute FS_{req}, yielded the same results as MC simulations using the same PDF data and a single random variable, R. This criterion is recommended as a preliminary substitute to full probabilistic MC analysis that may therefore be avoided within the scope of the gravity dam stability problem studied herein. The proposed simplified approach yields a good accuracy with a 10% maximum difference with respect to a more comprehensive MC reference solution. The criterion "$AFS \geq FS_{req}$", which introduces an additional safety margin, cannot be compared the MC reference solution, because no additional safety margin was introduced in the MC solution. "$AFS \geq FS_{req}$" yielded a lower allowable water level (83 m) than the MC solution (90 m). However, $H_w = 83$ m does correspond to an established rule of thumb to estimate the capacity of a 80 m-high gravity dam.

- The proposed AFS method is a practical answer to the need for a simplified and robust method to introduce material data shear strength uncertainties using a rigorous approach in gravity dam stability analysis. Its range of applicability and the adequacy of the safety margin provided as compared to reference MC solutions make ~~RBSF~~ AFS a useful tool to use in sensitivity analysis of PDF data before undertaking more comprehensive MC analyses (or variants such as FORM).

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Appendix 1 – ICOLD Benchmark material data

The fifteen couples of friction angle ϕ (°) and cohesion C (kPa) specified as input "material test" data in the 11th ICOLD Numerical Benchmark (Escuder-Bueno, Altarejos-Garcia and Serrano-Lombillo 2011) are: (ϕ, C)={(45, 500); (37, 300); (46, 300); (45, 700); (49, 800); (53, 200); (54, 600); (45, 0); (49, 100); (60, 200); (63, 200); (62, 400); (60, 700); (56, 100); (62, 400)}.

Abstract

Structural sliding stability of gravity dams is most often quantified using a binary safe/unsafe criterion such that the shear resistance has to be larger than the driving shear load. Large deterministic factors of safety, FS_{det} (e.g. 3 in normal condition), are used in existing guidelines to guard against material and loads uncertainties. Some guidelines allow an arbitrary reduction in FS_{det} (e.g. 2) when the knowledge in strength parameters increases from material test data. Yet, those reduced FS_{det} are not based on a rational consideration of uncertainties. Propagation of uncertainties could be done using comprehensive probabilistic analyses, such as Monte-Carlo simulations (MC). However, MC are complex and challenging for practical use. There is thus a need to develop simplified reliability based safety assessment procedures that could rationalise the adjustment of FS_{det} from existing dam safety guidelines. This paper presents a progressive analysis methodology using four existing safety evaluation formats of increasing complexity: (i) deterministic, (ii) semi-probabilistic (partial coefficient), (iii) reliability based *Adjustable Factor of Safety (AFS)*, and (iv) probabilistic (MC). Comprehensive comparisons are made for the sliding safety evaluation of a 80 m gravity dam. Results are presented in terms of sliding factors of safety, allowable water levels, and demand/capacity (load and resistance) ratios. It is shown that the reliability based AFS formulation, using direct integration, is simple and practical to use in complement to existing dam safety guidelines before undertaking MC simulations. AFS yielded results with a maximum difference of approximately 10% as compared to rigorous MC probabilistic analyses.

1. Introduction

Dams are major infrastructures, for which failure is an extremely rare event but with very serious consequences. Gravity dams are particularly sensitive to overtopping, due to the large hydrostatic thrust and uplift pressures that reduce the shear strength capacity that could be mobilised by frictional resistance and cohesion along lift joints or the concrete-rock interface. Dams have a useful service life extending for decades, such that aging of materials and the magnitude of anticipated floods evolve with advances in predictive methodologies and changes in the environment. The structural stability of major dams needs to be re-evaluated every 5-10 years according to Hazard Classification Systems (HCS), most often within the legal framework of a governmental regulatory agency.

Structural stability against sliding should satisfy a binary safe/unsafe limit-state stating that the shear resistance, R, has to be strictly larger than or equal to the driving shear load, L. To guard against uncertainties in R and L, large deterministic factors of safety (FS_{det}) are used. These large FS_{det} may be reduced when new knowledge about the material shear strength parameters is acquired to better quantify the friction coefficient and cohesion. For instance, in CDA (2007), $FS_{det} = 3$ if no material test is available, and $FS_{det} = 2$ if tests are done. A better knowledge of strength parameters thus authorises a reduced safety margin. However, the specified FS_{det} numerical values ~~come~~ are rounded numbers from experience and not from a rational approach to quantify reduction in strength parameters uncertainties. There is thus a need to develop, validate and verify simplified reliability based safety assessment procedures that could rationalise the FS_{det} adjustment ~~from existing dam safety guidelines, used routinely by practicing engineers~~ by referring to data bases developed from evaluating L and R uncertainties readily available from countless test series and load statistics (Kreuzer and Léger 2013). Simplified methods should be robust, computationally efficient, with a limited number of random variables, and easily interpretable.~~, and should be integrated as an extension of existing guidelines.~~ They then can replace the deterministic L and R parameters. Moreover, reliability based safety assessment ~~should~~ will also answer a societal aspect in form of relating the required safety margin to a HCS used by the dam safety regulations of the particular country~~. need of transparency, and be communicated to decision makers and stakeholders with clarity.~~ Of course, comprehensive reliability based probabilistic analyses could be undertaken to propagate uncertainties in structural stability analyses. However, comprehensive probabilistic analyses (e.g. Monte Carlo (MC) simulations) require extensive time, sophisticated tools, expertise and resources that are most often not available. To adjust the required safety margin as a function of a reduction in shear strength uncertainties, this paper presents a progressive analysis methodology of increasing complexity and accuracy. It consists in applying successively (i) deterministic analyses, (ii) simplified reliability based *Adjustable Factor of Safety (AFS)* analyses, and (iii) full probabilistic analyses (crude MC). The main objective of the paper is to establish the range of AFS modelling parameters for which coherent results could obtained with MC simulations that are used as reference values.

This paper is organised as follows. After the review of literature in section 2, section 3 describes four different safety formats: (i) deterministic, (ii) semi-probabilistic (partial coefficient), (iii) reliability based AFS, and (iv) probabilistic (MC). Comprehensive comparisons of the four safety evaluation formats are done by comparing sliding stability indicators of a 80 m-high gravity dam, studied in a previous ICOLD numerical Benchmark seeking to quantify the sliding probability of failure, p_f, as a function of the upstream reservoir water elevation, H_w (Escuder-Bueno et al.~~, Altarejos-Garcia and Serrano-Lombillo~~ 2011). It is shown that coherent results, as compared to MC, ~~are~~ could be obtained while using the AFS ~~computation~~ method when the required safety margin is being compared to the ratio of resistance and load mean values ~~as compared to MC~~. The AFS method is thus shown as a simple and practical preliminary substitute to full probabilistic MC analysis that may therefore be avoided within the scope of the gravity dam stability problem studied herein. Two additional AFS safety margin formulations have also been investigated to demonstrated the AFS range of applicability as compared to MC (i) when all uncertainties have been factored in AFS parameters such that AFS ≥ 1, and (ii) when an additional safety margin is deemed necessary to account for ignorance, or lack of information in spite of having considered uncertainties with all AFS factors (Kreuzer and Leger 2013). ~~in a much simpler and practical way.~~

2. Stability assessment of gravity dams considering uncertainty analysis

One can solve the gravity dam stability problem as a deterministic one where there is no uncertainty. At the other end of the spectrum, total ignorance of phenomena that may affect stability, and its outcome, cannot be quantified because classification and identification of what is unknown is not possible (Riley, Webley and Thomson 2017; D. Rumsfeld famous deep "*unknown-unknown*" uncertainty; Baecher 2016). We will therefore focus our analysis on epistemic and aleatory uncertainties in input data (friction coefficient and cohesion) that admit statistical descriptions from a frequentist point of view.

Deterministic analysis is traditionally used to assess the stability of dams (ANCOLD 2013; CDA 2007; FERC 2002; Ruggeri 2004; USBR 1976; USACE 1995, 2005). Deterministic methods do not allow accounting explicitly for uncertainties in strength capacity and load effects (stresses). Moreover, deterministic FSs are round numbers without rational safety-relevant significance, apart from grading them to the frequency of load combination (usual, unusual, extreme).There is thus a huge interest in the profession to move towards more refined ~~probabilistic~~ methods to consider uncertainties.

Probabilistic analysis allows ~~quantifying engineering judgement~~ considering nature's randomness, and human incomplete knowledge within the probabilistic safety evaluation. Mathematical constructs are used normally presented by probability density functions and engineering judgment. Then, comparisons between several failure scenarios ~~options~~ are possible while assessing probability of failure, p_f, and related risk according to the consequence model. Probabilistic methods are thus very useful tool to decision-making (Bury and Kreuzer 1985; FERC 2014). Probabilistic risk analysis, and associated risk management, are used for other civil structures and are now in active development in dam engineering (Baecher 2016; FERC 2014; Hartford and Baecher 2004; Hartford et al. 2016; Kalinina et al. 2016; Peyras et al. 2010; Peyras et al. 2012; SPANCOLD 2013; Westberg Wilde and Johansson 2016; Zhang et al. 2016). As dam engineering is moving towards models accounting for uncertainties, ICOLD organized an International Benchmark Workshop on Numerical Analysis of Dams in 2011 to examine probabilistic analysis (Escuder-Bueno et al.~~, Altarejos-Garcia and Serrano-Lombillo~~ 2011). A new ICOLD Benchmark on Risk Assessment ~~will~~ took~~ake~~ place in 2017 (Johansson et al. 2017). Risk assessment uses probabilistic methods, and consists (i) to combine failure modes to determine a failure scenario by using event or failure trees~~; a tree of events is often used~~, (ii) to model ~~each~~ failure modes, (iii) to ~~compute~~ assess the failure probability for each failure mode, (iv) to deduce the total dam failure probabilities ~~for the whole scenario~~. The concept of "*As Low As Reasonably Possible (ALARP)*" is then used in several dam safety guidelines to answer the difficult question "*How safe is safe enough?*" (Bowles 2007). ALARP is used to accept or reject the resulting failure probability and related consequences. Recognising that absolute safety cannot be achieved, the mathematical concept of risk is used to specify reasonable efforts to avert losses considering what can be achieved with the available resources. Reasoning in terms of probability of failure and acceptable risks is a key societal construct, subjected to time evolution. While accepting the ALARP principle, it is then possible to make the transition between the deterministic world, where achieving the required FS_{det} is believed to provide absolute safety, and the probabilistic world were uncertainties are explicitly recognised (Luhmann 2005).

A probabilistic analysis could account for uncertainties in cohesion, friction, drain efficiency, and several other parameters as shown in Altarejos-Garcia et al. (2015, 2012). Probabilistic analysis also allows reliability assessment of dam-foundation-reservoir systems (Westberg Wilde and Johansson 2013). In ~~probabilistic~~ reliability-based safety assessment, fragility analysis, first used in seismic evaluation of nuclear power plants, is a key step (Porter 2017). Fragility analysis is the computation of the probability to reach an undesirable limit-state for a known loading intensity. Fragility curves, expressed as function of reservoir elevation, H_w, provide quantitative cumulative distribution functions for the dam to resist sliding. ~~Fragility functions are used in damage analysis that is combined with loss analysis to quantify risk (Porter 2017)~~. Fragility curves provides a rational tool that could be used to compare several remedial options if a need for strengthening is identified (Ebeling et al. 2012; Ellingwood and Tekie 2001; Tekie 2002).

However, probabilistic assessments require many parameters to describe uncertainties, such as random variables, Probability Density Functions (PDF), PDF bounds, coefficients of variation, which may affect substantially the analysis results as shown by the wide dispersion in fragility curves computed from the participants in the ICOLD Benchmark (Escuder-Bueno et al. 2016). Then, the decision to take remedial action, if necessary, depends on a complex and possibly not unique assessment: one may wonder about limits of reliability methods, which are finally not the unerring reference (Kreuzer 2000, 2003).

Semi-probabilistic (partial coefficient) analysis is a first simplified approach that allows considering uncertainties according to each load and resistance parameter (ANCOLD 1991; CFBR 2013, 2015; IS 1984-1998; Peyras et al. 2008; Rocha 1974; SPANCOLD 2003). Partial safety coefficients are ideally calibrated according to probabilistic analyses. However, each dam is a unique hydro-geomechanical system. Calibration may not be based on consistent sampling, but adapted to correspond to existing structures designed with deterministic methods (Jongejan and Calle 2013; Kovarik 2000). In the Netherlands, the National Flood Risk Analysis project developed recommendations evolving from deterministic to semi-probabilistic analysis to assess stability of hydraulic structures (levees), using probabilistic analysis to calibrate coefficients according to target failure probability, p_f* (Jongejan and Maaskant 2013; Vergouwe 2016).

Adjustable Factor of Safety (AFS) analysis is another simplified and practical way to perform probabilistically (reliability) based safety assessment. AFS considers only two basic random variables, the resistance **R** and the load, **L**. AFS is seeking a binary outcome such that AFS $\geq FS_{req}$, where FS_{req} is depending on a user defined target p_f* or a target reliability index, β*. As described in more details in section 3, AFS connects R and L within a probabilistic framework using their PDF along with six uncertainty factors calibrated on empirical evidences, including coefficients of variations as well as upper (for L) or lower (for R) PDF bounds (Kreuzer and Léger 2013). ~~More comprehensive probabilistic analyses may attempt a solution requiring much more extensive numerical data processing. However, no matter how mathematically elaborate is a probabilistic analysis, it cannot avoid the key difficulty of acquiring data to characterise each random variable from reliable empirical evidences.~~

A progressive approach, with increasing level of complexity to assess the sliding stability of gravity dams is therefore proposed as follows: (i) deterministic analysis is done first, (ii) then semi-probabilistic method can optionally be used to distinguish uncertainties about the friction coefficient and cohesion. (iii) The AFS method is next performed to rationally adjust the computed FS_{det} and compared it to FS_{req} considering a probabilistic description of R uncertainties and a selected target p_f* or β*. Finally, (iv) full probabilistic analysis may be undertaken if deemed necessary.

3. ~~Quantitative~~ Specification for required~~ments~~ ~~in~~ safety margins in investigated safety evaluation formats

3.1 Deterministic

Deterministic analysis is traditionally used to design and assess dam stability. It consists in defining a factor of safety (FS) between the dam's resistance R and the loading L from $FS = R/L$, and to compare it to required values according to applicable guidelines. The gravity method and Mohr-Coulomb criterion are most often employed to define the shear strength resistance to compute sliding FS. Typically, three values for resistance are considered in parametric study: best estimate, lower bound, and upper bound. Different load combinations are considered, only those associated with water levels are studied herein: usual, unusual, and extreme (flood). Moreover, required FS relate to dam-foundation interface and lift joints. ~~These coefficients come from experience and have no sound mathematical justification.~~

Some guidelines require FS without any consideration to the level of knowledge about strength parameters (Table 1). Some other guidelines recommend FS considering the level of knowledge in strength parameters. The required FS are larger if no material tests have been realised (Table 2).

Table 1. Required deterministic factors of safety without any explicit consideration of uncertainties.

Load combination	**Required factor of safety**		
	UBSR (1976)	USACE (1995)	FERC (2002) [(a)(b)]
Usual	3.0	2.0	3.0
Unusual	2.0	1.7	2.0
Extreme	1.0	1.3	-

[(a)] Dams are categorised according to the consequences of a failure: the coefficients are for a dam with moderate to high risks.
[(b)] Other FS are required for friction only (cohesion is null).

Table 2. Required deterministic factors of safety depending on the level of knowledge of strength parameters.

Load combi-nation	**Required factor of safety**					
	CDA (2007) [(a)]		ANCOLD (2013) [(a)]		USACE (2005) [(b)]	
Knowledge?	*No tests*	*With tests*	*Not well-defined* [(c)]	*Well-defined* [(c)]	*Well-defined* [(d)]	*Ordinary*
Usual	3.0	2.0	3.0	2.0	2.0	1.7
Unusual	2.0	1.5	2.0	1.5	1.5	1.3
Extreme	1.3	1.1	1.5	1.3	1.1	1.1

[(a)] Other FS are required for friction only (CDA 2007) or residual values for C, tanϕ (ANCOLD 2013).

[(b)] Dams are categorised according to the consequences of a failure: the FS are for a dam with moderate to high risks.
[(c)] According to ANCOLD (2013), "well-defined" means that "a sufficient number of tests have been done to specify the strength parameters with reasonable certainty (e.g. assumed strength is exceeded by 80% of the test results from a test regime involving a significant number of tests)".
[(d)] According to USACE (2005), site information is "well-defined" when records are available, dam is monitored, uplift are known, and "foundation strengths can be established with a high level of confidence".

3.2 Semi-probabilistic (partial coefficients)

Rocha (1974) suggested to introduce partial safety coefficients for friction and cohesion instead of a single FS. This approach is to account for different levels of uncertainties in these two shear strength mechanisms. Divisor coefficients of 1.5 to 2, and 3 to 5, could be applied respectively to the friction coefficient, tanϕ, and the cohesion, C. It is thus recognised that uncertainties in cohesion are more important than in friction, whereas there is no mathematical justification for the recommended values. Thereafter, countries like India (IS 1984-1998) and Spain (SPANCOLD 2003) chose to use the concept of partial strength reduction coefficients (Table 3).

Semi-probabilistic assessment consists in using some partial safety coefficients for both loads and strengths, increasing loads and reducing strengths depending on the target reliability of the structural component. These coefficients are ideally calibrated from probabilistic methods, and adapted to existing structures designed with deterministic analysis (Jongejan and Calle 2013; Kovarik 2000).

Table 3. Semi-probabilistic partial safety coefficients

Load combination	**Partial safety coefficients**							
	IS (1984-1998)		SPANCOLD (2003) [(b)]		CFBR (2013)		ANCOLD (1991) [(c)]	
	γ_C [(a)]	$\gamma_{\tan\phi}$	γ_C	$\gamma_{\tan\phi}$	γ_C	$\gamma_{\tan\phi}$	Strength	Load [(d)]
Usual	3.6	1.5	5.0	1.5	3.0	1.5	0.3	
Unusual	3.6	1.5	4.0	1.2	2.0	1.2	0.4	

Extreme – flood	1.2	1.0	3.0	> 1.0	1.0	1.0	0.8

[(a)] C is the cohesion, $\tan\phi$ is the friction coefficient; γ_C and $\gamma_{\tan\phi}$ are associated partial safety coefficients such that semi-probabilistic sliding stability limit-state criterion is: $\frac{A_c.C/\gamma_C+V.\tan\phi/\gamma_{\tan\phi}}{L} > 1.0$ where V is the sum of vertical forces, A_c the compressed sliding area, L the hydrostatic thrust (IS 1984-1998; SPANCOLD 2003; CFBR 2013).
[(b)] Dams are categorised according to the consequences of failure: the coefficients are for a dam with moderate to high risks.
[(c)] Multiplier coefficients applied to strength and load parameters to compute R′ and L′ such that semi-probabilistic sliding stability limit-state criterion is: $R' > L'$ (ANCOLD 1991).
[(d)] Coefficients for loads are 0.95 for water and well-known dead loads contributing to stability, 0.90 for not well-known concrete weight, 1.05 for water, uplift and dead loads contributing to instability, 1.50 for live and silt loads contributing to instability.

3.3 Probabilistic

Deterministic FS and partial safety coefficients do not inform about the safety margin including uncertainties about parameters employed in analysis. Probabilistic analysis allows to compute a failure probability, p_f, considering mathematically uncertainties. HCS have been developed to differentiate structures according to the consequences a failure. The ALARP concept has often been used to link loss of human lives (or persons in danger), more than material costs of consequences, to recommended acceptable failure probability (Figure 1).

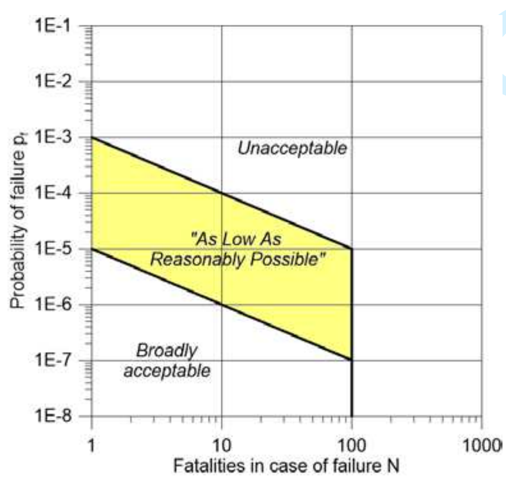

Figure 1. Concept of "As Low As Reasonably Possible" adapted from (CDA 2007).

In probabilistic analysis, load (L) and resistance (R) parameters are statistically distributed according to PDF selected to be representative of tests and knowledge about these parameters. Failure develops when internal load demand exceeds the resistance capacity of the dam. Influences of different selections of PDF data in probabilistic analyses have been studied in Altarejos-Garcia et al. (2012); Carjaval et al. (2009a, 2009b, 2009c); Carjaval, Peyras, and Baconnet (2010); Krounis and Johansson (2012); Krounis et al. (2016); Lombardi (1988, 1993, 2006); Spross, Johansson and Larsson (2013). Probabilistic analysis, no matter how sophisticated, can still lead to very different solutions for a given problem because of the complex choices of random variables, characteristic values, PDF, bounds, which can largely influence final results. For example, Figure 2 illustrates the wide dispersion obtained from participants in an ICOLD Benchmark seeking to compute the fragility curve (Fcurve) for p_f as a function of H_w for a 80 m-high gravity dam given fifteen sets of "cohesion, friction angle" data pair representing material test data. The detailed description of the problem is given in section 4.1 of this paper. In Figure 2, we present results from all participants that used MC simulations, (Fcurves 1-7) as well as our own MC solutions (Fcurves 8-12). Using the same PDF data for C and tanϕ, ICOLD Fcurve 2 is similar to Fcurve 8 computed herein for water levels ranging from 75 m to 80 m. Our MC probabilistic analysis procedure is thus validated for the selected PDF data.

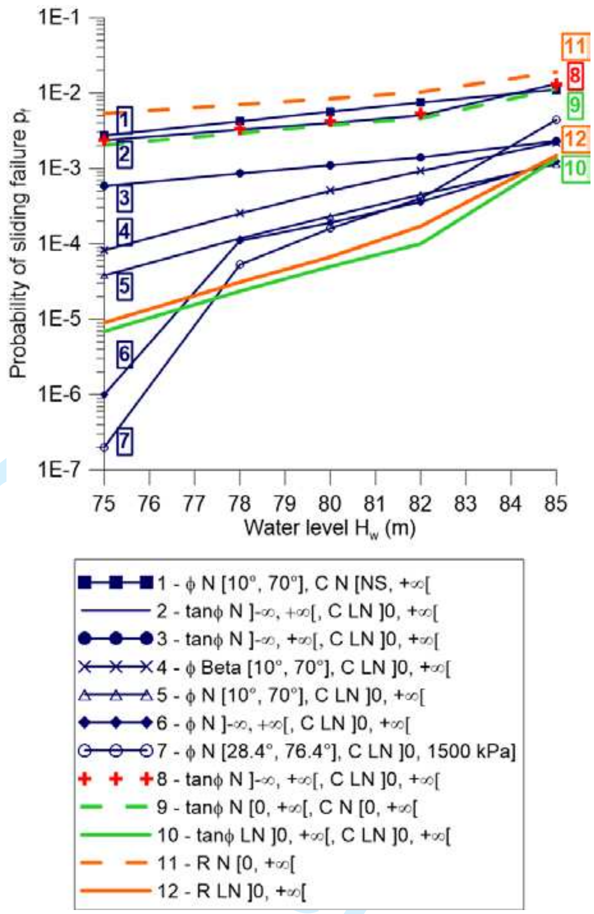

Figure 2. Dispersion in fragility curves computed for the ICOLD Benchmark for a 80 m-high gravity dam (Fig. 5); ϕ is the friction angle, C the cohesion, R the global resistance of the dam defined in Eq. 2; N = normal distribution, LN = lognormal distribution; selected bounds are indicated. Fcurves1 to 7 are from participants in the ICOLD Benchmark, Fcurves 8 to 12 are our solutions to the Benchmark.

3.4 Reliability based Adjustable ~~safety~~ Factor~~s~~ of Safety: AFS

Kreuzer and Léger (2013) presented a simplified reliability based method to assess dam stability. It depends on two uncertain random variables, R and L. An *Adjustable Factor of Safety (AFS)* is defined by:

$$AFS = \frac{\mu_R\{1-(k_R.c_R+\alpha_R)\}}{\mu_L\{1+(k_L.c_L+\alpha_L)\}} = FS_{det}.U_{RL} \qquad \text{(Eq. 1)}$$

FS_{det} is the deterministic factor of safety, c_R and c_L are the coefficients of variation related to physical uncertainties, the natural intrinsic dispersion of values, k_R and k_L are related to statistical uncertainties defined herein as the lack of knowledge, from the number and reliability of material test data and α_R and α_L are related to model or (epistemic) uncertainties (Figure 3). Comprehensive description and numerical values for these coefficients have been suggested in Kreuzer and Léger (2013) depending on the knowledge of the structure.

Considering only two random variables, R and L, the AFS aims to be compared to a Required Safety Factor, FS_{req}, depending on a target failure probability, p_f*, or the corresponding reliability index β*. For a safe structure, the stability criterion becomes AFS ≥ FS_{req}. FS_{req} is computed iteratively by direct integration to be the FS = μ_R/μ_L such that with the selected PDF data for R and L, the computed p_f would correspond to the target failure probability, p_f*. PDF are bounded at distances from the mean corresponding to a number k_R or k_L of standard deviations, on the left for the resistance and on the right for the load. For unbounded PDF, tails of distributions are considered in the computation of FS_{req} (it would correspond to $k = \infty$), but a k value has to be defined for the computation of AFS. FS_{req} decreases when the uncertainties in L and R are reduced. The AFS is a simple approach to introduce uncertainties using PDF data of the basic random variables **L** and **R** directly into the safety evaluation process. It allows to study the effect of reducing the coefficient of variation of the resistance, $c_{\mathbf{R}}$, using material tests, on a rational basis as opposed to existing deterministic dam safety guidelines using arbitrary reduced FS requirements. The AFS considers only p_f*, that could be specified directly from a HCS, avoiding to rely on assessing speculative terms of risk which require a complementary loss model. A user friendly open source computer program, **R-AFS**, was developed to perform AFS and FS_{req} computations (Morin 2016). The R-AFS implementation is controlled by an input-output environment, using the "**R**" open-source statistical computational platform (see https://www.r-project.org/). A copy of R-AFS could be obtained by contacting the second author (pierre.leger@polymtl.ca).

~~While discussing AFS with colleagues, we realized that the expression "*Adjustable*" is unfortunate because for some it conveys the impression of an *arbitrary* adjustment instead of a reliability based adjustment using sound principles. We are thus using below the expression Reliability Based Safety Factor (RBSF). RBFS is, we believe, more suitable to convey the essence of the proposed simplified method.~~

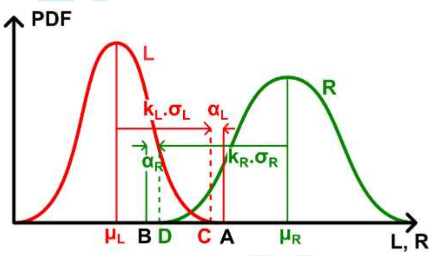

Figure 3. Definition of the six uncertainty coefficients in the reliability-based AFS format: (i) $c_R = \sigma_R/\mu_R$, (ii) k_R, (iii) α_R, (iv) $c_L = \sigma_L/\mu_L$, (v) k_L, (vi) α_L.

The advantages of the reliability-based AFS are (i) the rationality to account for uncertainties using selected PDF data and a target p_f* (or β*) in similarity to probabilistic analysis, (ii) its simplicity and practical use, (iii) a clear interpretation in the form of a binary decision to accept/reject the computed FS.

Of course, if one has the certitude to have properly factored all uncertainties with the c, k and α values, $FS_{req} = 1$ would be adequate. The general accepted safety performance criterion then becomes $AFS \geq 1$. Another approach is to consider all uncertainties in the computation of FS_{req}. In the gravity dam shear strength problem, we are then seeking to satisfy $(\mu_R/\mu_L) \geq FS_{req}$ (c_R, k_R, α_R, p_f*). However, an additional safety margin might be provided for initial imperfection, ignorance or lack of information leading to the acceptance criterion AFS $\geq FS_{req}$ (c_R, k_R, α_R, p_f*) (Kreuzer and Léger 2013). These three acceptance criteria (i) AFS ≥ 1, (ii) $(\mu_R/\mu_L) \geq FS_{req}$, and (iii) AFS $\geq FS_{req}$ are compared with MC analyses, used as the reference solution to evaluate the reliability-based AFS ~~RBFS~~ method.

3.5 Progressive approach to introduce uncertainties

The above safety evaluation formats, ranging from deterministic to comprehensive probabilistic analyses, show different ways to account for uncertainties, from various sources appearing at each level of the stability assessment of the dam: material testing, selection of strength and load parameters, structural model. A progressive approach may then be developed to best account for these uncertainties, from simple to more complex but more precise evaluation formats (Figure 4).

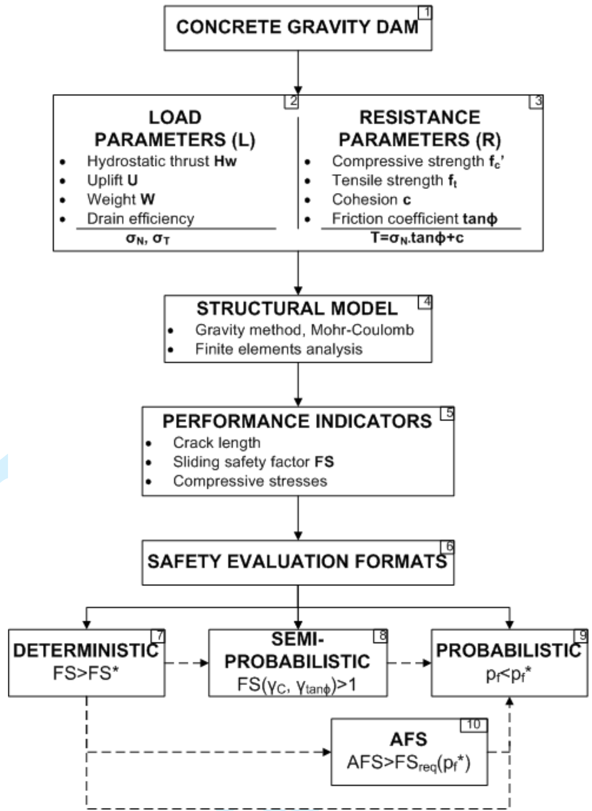

Figure 4. Progressive approach for dam safety assessment: parameters, sources of uncertainties and performance indicators.

4. Application of progressive safety assessment

4.1 Description of the gravity dam for applications

The dam for applications is a 80 m-high concrete gravity dam that might be subjected to overtopping. The dam geometry (Figure 5) is given in the 11th ICOLD Numerical Benchmark (Escuder-Bueno et al.~~, Altarejos-Garcia and Serrano-Lombillo~~ 2011). A single and simple failure mode, corresponding to horizontal sliding along the dam-foundation interface, is to be investigated. In reality, the kinematic of a dam base sliding-turning failure mode might occur along inclined planes propagating in the foundation (Fishman 2009). However, our study is restricted to the ICOLD benchmark problem to allow comparisons with previously published results (Fig. 2). The resistance, **R**, is a function of two basic random variables, (i) the friction coefficient, tanϕ, and (ii) the cohesion, C. The dam weight, W, and the drain effectiveness, E, are considered as given constant parameters. The uplift pressure, U, is a function of the water level, H_w. In this application, there is no uncertainty for the load, **L**. The water level, H_w, is increased systematically to reach an unacceptable limit state. In the case of overtopping, the water weight on the crest is estimated as W_w. The gravitational acceleration, the dam-foundation interface tensile strength, the water and concrete densities used in computations are respectively, $g = 9.81$ m/s², $f_t = 0$, $\rho_w = 1000$ kg/m^3, and $\rho_c = 2400$ kg/m^3. The classical gravity method is used in stability analyses, considering cracking at the dam-foundation interface. If the base crack extends beyond the drain, the full uplift pressure is considered in the crack.

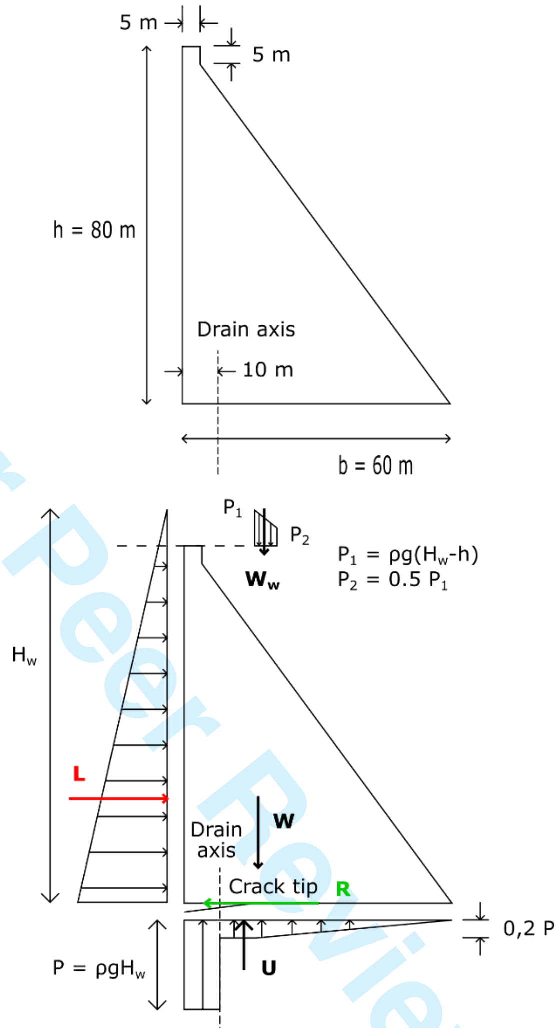

Figure 5. Geometry, load, L, drainage, and resistance, R, (friction and cohesion) properties of the gravity dam analysed.

Fifteen couples of friction angle ϕ (°) and cohesion C (kPa) are specified as input "material test" data in the 11th ICOLD Numerical Benchmark seeking to estimate the sliding probability of failure, p_f, of the dam (see Appendix 1). The statistics for C and the friction coefficient, tanϕ, are summarised in Table 4. The coefficient of variation for cohesion, c_C, is 0.67, which is quite large. Distribution fitting has been realised with N-PDF and LN-PDF. The LN-PDF was found to suit the data best taking into account the skewness, whereas N-PDF is symmetrical.

Table 4. Material test data statistics for friction and cohesion at the dam-foundation interface.

Material test	Cohesion C (kPa)	Friction angle ϕ (°)	Friction coefficient tanϕ
Mean μ – best estimate	367	52.4	1.36
Standard deviation σ	247	7.99	0.39
Coefficient of variation c	0.67	0.15	0.29
Minimum – lower bound	0	37	0.75
Maximum – upper bound	800	63	1.96
5% fractile – N	0	39.3	0.72
5% fractile – LN	112	40.3	0.82

4.2 Deterministic stability evaluation

Deterministic analyses are first realised with mean values selected as best estimates for cohesion, C, and friction coefficient, tanϕ. Smallest and largest values are taken as lower and higher bounds. For the usual and unusual load combinations, the maximal allowable water level, H_w, is computed according to CDA (2007, Table 2), without and with material tests (Figure 6).

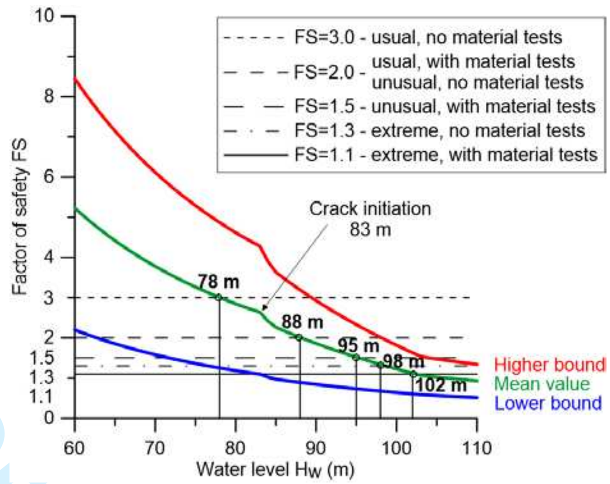

Figure 6. Maximum allowable water level according to deterministic CDA (2007) dam safety guidelines.

These results indicated the importance of having a good knowledge of shear strength parameters. For the usual load combination, the allowable water level increased by 10 m if material tests are realised. However, CDA (2007) does not provide clear guidance on the number of tests, the sampling location and the testing method to be used to obtain representative results with a quantified confidence level. ANCOLD (2013) suggests criteria for the "well-defined" material shear strength parameters (Table 2).

4.3 Probabilistic safety evaluation using Monte-Carlo simulations

A probabilistic assessment requires to select a target p_f*, random variables, their PDF, and their bounds if they are bounded. PDF are bounded at distances from the mean corresponding to a number *m* of standard deviations on the left and on the right. Unbounded PDF corresponds to $m = \infty$. Herein, the p_f* is 10^{-5}, consistent with the ALARP principles for a high-risk dam (CDA 2007). We work with two sets of random variables either (C, tanϕ) or R. The resistance, R, is described as the sum a friction and a cohesion component:

$$R = V.tan\phi + A_c.C \qquad \text{(Eq. 2)}$$

a mean value is computed for R and the standard deviation, σ_R, is estimated from:

$$\sigma_R = \sqrt{\left(V.\sigma_{tan\phi}\right)^2 + (A_c.\sigma_C)^2} \qquad \text{(Eq. 3)}$$

where V is the sum of vertical forces, and A_c the area in compression.

The related PDF are successively selected as N and LN in sensitivity analyses. Bounded and unbounded PDF are also studied. MC computations are realised with MATLAB® (The MathWorks 2016), $n = 10^7$ samples are found adequate to obtain convergence for p_f. For instance, when $p_f = 10^{-5}$, the accuracy is $p_f = 10^{-5} \pm 5.10^{-7}$.

Unbounded PDF are first studied. When the unbounded hypothesis is considered with N-PDF, negative values of C, tanϕ, or R, are replaced by new draws in MC simulations. The results are presented in Figure 7. LN-PDF yielded failure probabilities smaller than N-PDF. The reduction from two random variables, (C, tanϕ), to one random variable, R, gave similar H_w results.

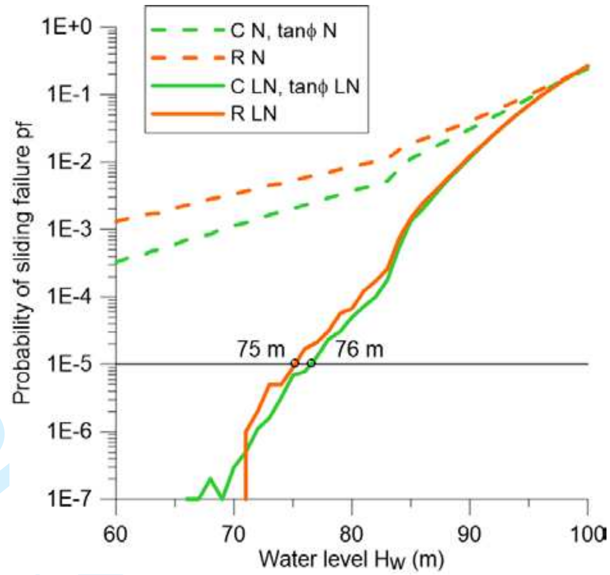

Figure 7. Fragility curves comparing **unbounded** N-PDF and LN-PDF, variables (C, tanϕ) or R, computed with MC; and H_w according to probabilistic analysis for a target failure probability $p_f* = 10^{-5}$.

The effect of ***bounded PDF*** is investigated by selecting values between the 5% fractile for strength parameters on the left of the distribution: m_l standard deviations, and the 95% fractile on the right: m_r standard deviations (Figure 8). For variables C, tanϕ, R, m_l are respectively equal to 1.03, 1.38, 1.39 and m_r respectively equal to 1.88, 1.84, 1.83.

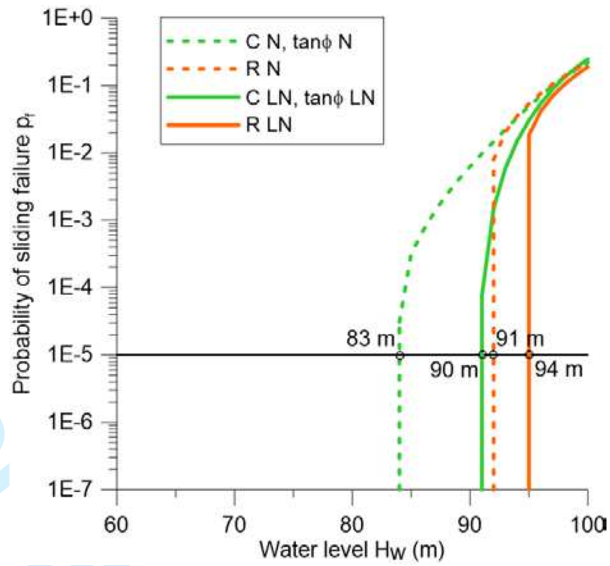

Figure 8. Fragility curves comparing **bounded** N-PDF and LN-PDF at the 5% and 95% fractile values, variables (C, tanϕ) or R, computed with MC; and maximum allowable water level according to probabilistic analysis for a $p_f^* = 10^{-5}$.

Results were similar using N-PDF or LN-PDF, but more sensitive to the selection of random variables. Using a single random variable, R, instead of two (C, tanϕ), yielded higher H_w.

4.4 Semi-probabilistic (partial coefficient) stability evaluation

For the semi-probabilistic analysis, CFBR (2013) suggests as characteristic values, a "wise estimation of the mean", and the 5% fractile if statistical methods are used. Herein, two pairs of (C, tanϕ) are used (i) mean values as best estimates (367, 1.36), and (ii) 5% fractile obtained from the 15 material test data assuming a N-PDF (0, 0.72, Table 4) considered in typical user of CFBR. The partial strength safety coefficients are applied for the usual, unusual, and extreme load combinations (Table 3).

With the extreme combination, the computed H_w was 106 m using the mean values for (C, tanϕ), but as complete base cracking occurred at **104 m** we used this last value as the maximum H_w. Allowable H_w according to each load combination are presented in Figure 9.

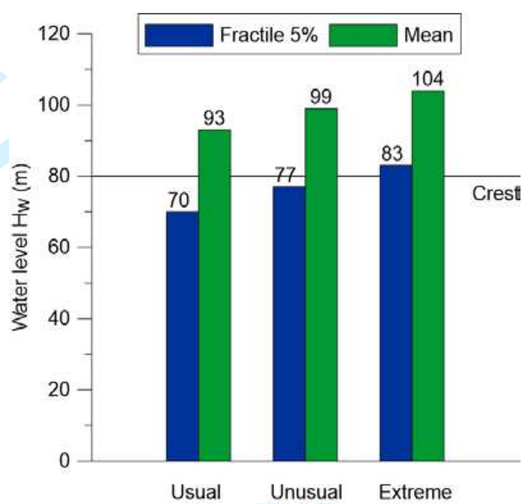

Figure 9. Maximum allowable water level according to semi-probabilistic CFBR (2013) dam safety guidelines.

These results showed a very significant sensitivity of the semi-probabilistic method to the selected characteristic values.

4.5 Reliability-based Adjustable Safety Factors – AFS

The selected p_f* is 10^{-5} as recommended for a high-risk dam with good quality assurance and management (Kreuzer and Léger 2013). The AFS method is employed without uncertainties in load L, $c_L = 0$, also, coefficients reporting model uncertainties α_R and α_L

are null. The shear strength random variable in the AFS method is R. The related PDF is LN as recommended in Kreuzer and Léger (2013).

Unbounded PDF are first studied. Tails of distributions are considered while computing FS_{req} but a value for k_R has to be defined for the evaluation of AFS. Values selected for k_R in the AFS computation (Eq. 1) are (i) $k_R = 1.39$, corresponding to the 5% fractile for R for LN-PDF (Holický 2009), (ii) $k_R = 2$, (iii) $k_R = 3$. FS_{req} is computed with unbounded PDF. Computed AFS and FS_{req} are presented in Figure 10. For $k_R = 3$, allowable water level was less than 40 m.

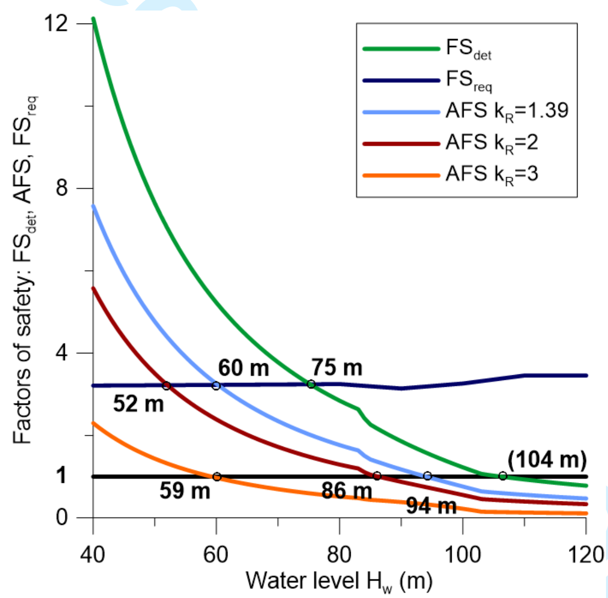

Figure 10. ~~RB~~AFS method: deterministic FS_{det}; required FS_{req} for $p_f^* = 10^{-5}$ and **unbounded** LN-PDF for R; and AFS for (i) $k_R = 1.39$, (ii) $k_R = 2$, (iii) $k_R = 3$.

For **bounded PDF** the effect of bounds is investigated by selecting (i) $k_R = 1.39$ (corresponding to the 5% fractile for R for our LN-PDF (Holický 2009), (ii) $k_R = 2$, and (iii) $k_R = 3$ for the computation of AFS as well as FS_{req}. The computed AFS and FS_{req} are presented in Figure 11. For $k_R = 3$, H_w was again less than 40 m.

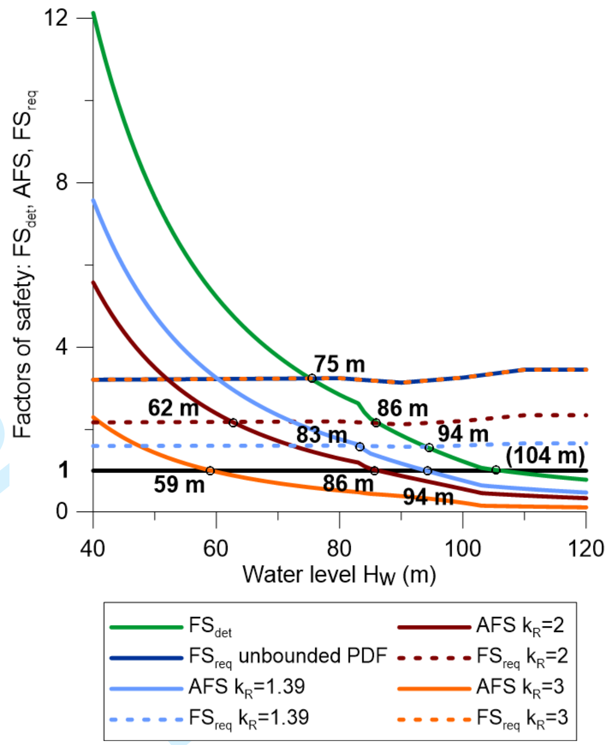

Figure 11. ~~RB~~AFS method: deterministic safety factor FS_{det}; required safety factor FS_{req} for $p_f^* = 10^{-5}$ and **bounded** LN-PDF for R; AFS and FS_{req} for (i) $k_R = 1.39$, (ii) $k_R = 2$, (iii) $k_R = 3$.

5. Discussion

5.1 Results from different safety evaluation formats

The 80 m dam was analysed according to four safety evaluation formats applying the proposed progressive safety assessment methodology. The key results are presented (i) in Figure 6 for deterministic analyses, (ii) in Figure 9 for semi-probabilistic analyses, (iii) in Figures 10-11 for ~~RB~~AFS, and (iv) in Figures 7-8 for probabilistic MC analyses. Comparative H_w results are presented in Figure 12. Table 5 presents the computed FS for each safety format that are compared to the required FS to declare a safe dam. The reference value to make comparisons and orient the discussion is $H_w = 90$ m. This allowable H_w is computed from MC simulations, using C and tanϕ as random variables, with LN-bounded

PDF (m_l = 1.03 for C and 1.38 for tanϕ, corresponding to the 5% fractile). It is a reasonable and defensible probabilistic model having considered strength uncertainty in a rational way with two random variables, as well as existing dam safety guidelines to select PDF bounds. Obviously, other reference value for H_w could be selected. However, we present coherent hypotheses moving from one level of complexity to the next such that meaningful comparisons and discussion could be established.

The ***deterministic*** format criteria (Table 2, CDA 2007) are unable to consider the large coefficient of variation in shear strength parameters, H_w being **102 m** for extreme conditions (flood) if material tests had been realised. A parametric analysis showed that lower bound of shear strength data would authorise H_w equal to **82 m**.

Probabilistic sensitivity analyses were applied with random variables (C, tanϕ) or R, LN-PDF, and unbounded or bounded distributions, with $p_f^* = 10^{-5}$. For the unbounded case, H_w was **76 m** with variables (C, tanϕ) and **75 m** with variable R, leading to similar results. With bounds corresponding to the 5% fractiles for strength parameters, H_w was **90 m** (the reference value) for variables (C, tanϕ) and **94 m** for R. Probabilistic analysis (MC) may be considered as the most rigorous approach but is shown to be sensitive to the selection of random variables and PDF bounds.

In ***semi-probabilistic analysis,*** two pairs of values for (C, tanϕ) were used. For the extreme combination, using the mean, H_w was **104 m.** Using the 5% fractile, H_w was **83 m**. This 83 m value was the same as using bounded N-PDF with variables (C, tanϕ) in probabilistic analysis (Figure 8), meaning that calibration of partial coefficients in semi-probabilistic analysis appears to be consistent with results of N-PDF bounded probabilistic analysis.

The ***reliability-based adjustable safety factors (AFS)*** with criterion "AFS $\geq FS_{req}$" yielded very low H_w for unbounded LN-PDF. For unbounded PDF, H_w values were very sensitive to the coefficient k_R. It is deemed inadequate in our application.

The criterion "AFS ≥ 1" gave the same H_w for unbounded and bounded LN-PDF because this criterion is not related to the computation of FS_{req}, using PDF data. For $k_R = 1.39$ and $k_R = 2$, H_w obtained with the criterion "AFS ≥ 1" are the same as those obtained with criterion "$(\mu_R/\mu_L) \geq FS_{req}$". The criterion "$(\mu_R/\mu_L) \geq FS_{req}$" gave the same H_w for unbounded PDF even for $k_R = 3$. This means that bounding PDF with large k_R is equivalent in ~~RB~~AFS to consider the whole content of the distribution.

5.2 Comparisons of different safety evaluation formats

For the same 80 m gravity dam stability problem, with known 15 pairs (C, tanϕ) and considering (i) no uncertainty, (ii) uncertainties believed to be known with certainty either in RBSF or probabilistic analysis or, (iii) uncertainties with an added safety margin, may decrease the allowable H_w from 104 m to 60 m (Figure 12). Table 5 presents FS computed by each safety evaluation format and the associated safety criterion for two water levels: 80 m and 90 m. Demand/capacity ratios (D/C) have been computed in each case as $(H_w^2/90^2)$ because the applied hydrostatic thrust, L, could be estimated as $L = (\rho_w\, g\, H_w^2)/2$. D/C ratios are presented in enclosed boxes in Figure 12, and for varying values of PDF bounds, m_l (MC) and k_R (~~RB~~AFS), in Figure 13.

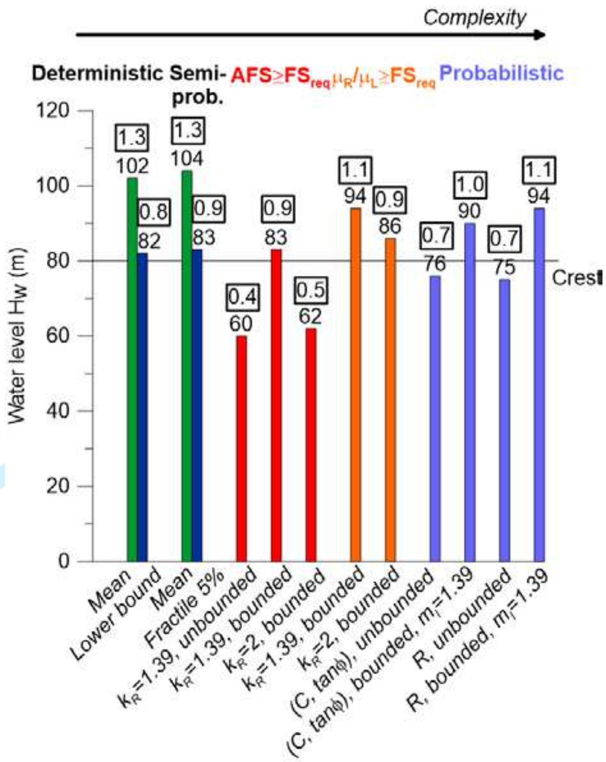

Figure 12. Maximum allowable water level for different safety formats: (i) deterministic, (ii) semi-probabilistic, (iii) AFS criterion "AFS ≥ FS_{req}", (iv) ~~RBSF~~ AFS criterion "(μ_R/μ_L) ≥ FSreq", (v) probabilistic MC simulations with random variables (C, tanϕ), (vi) probabilistic MC simulations with random variable R. LN-PDF is used for ~~RBSF~~ AFS and probabilistic methods. In boxes are Demand/Capacity ratios.

Table 5. Sliding FS from safety formats: (i) deterministic (CDA 2007), (ii) semi-probabilistic (CFBR 2013), (iii) three criteria of ~~RBSF~~ AFS method, (iv) probabilistic (MC simulations).

	80 m - unusual		**90 m - extreme**	
Deterministic $FSdet \geq FS_{det}^{req}$	FS_{det}	FS_{det}^{req}	FS_{det}	FS_{det}^{req}
	2.84	≥ 1.5	1.86	≥ 1.1
Semi-prob. $FS' \geq FS'^{req}$	FS'_{unus}	FS'^{req}	FS'_{extr}	FS'^{req}
	2.14	≥ 1.0	1.86	≥ 1.0
ASF [(a)] (i) $AFS \geq 1$	AFS	AFS^{req}	AFS	AFS^{req}
	1.77	≥ 1.0	1.17	≥ 1.0
(ii) $(\mu R/\mu L) \geq FS^{req}$	μ_R/μ_L	FS_{req}	μ_R/μ_L	FS_{req}
	2.84	≥ 1.61	1.86	≥ 1.58
(iii) $AFS \geq FS^{req}$	AFS	FS_{req}	AFS	FS_{req}
	1.77	≥ 1.61	1.17	< 1.58
Probabilistic [(b)] $FS_{pr} \geq FS_{pr}^{req}$	FS_{pr}	FS_{pr}^{req}	FS_{pr}	FS_{pr}^{req}
	1.26	≥ 1.0	1.0	≥ 1.0

[(a)] ~~RBSF~~ AFS computations with bounded PDF and $k_R = 1.39$.

[(b)] Reference value from probabilistic analyses is 90 m. FS_{pr} is defined by the inverse of the demand/capacity ratio.

Deterministic and semi-probabilistic formats do not allow to account for uncertainties in parameters used for computations. The allowable H_w were especially high using mean values as strength parameters (102 m and 104 m an allowable capacity approximately 30% larger than the reference value). Using lower bound or 5% fractile values as strength parameters in a sensitivity analysis yielded much smaller H_w (82 m and 83 m), because of the large scatter in test data.

Application of ~~RBSF~~ AFS method (computation of FS_{req} and AFS) allows to quantify uncertainties in the safety evaluation. Comparing (μ_R/μ_L) to FS_{req} is then a mathematically expressible safe/unsafe criterion. Allowable H_w were 94 m and 86 m for bounded PDF with k_R respectively equal to 1.39 and 2. This criterion yielded the same H_w as probabilistic MC simulations using variable R and bounded at 5% fractile. This is because direct integration used in computation of FS_{req} and MC simulations give the same p_f. While using $(\mu_R/\mu_L) \geq FS_{req}$, H_w is 94 m corresponding to a D/C of 1.1 using the 90 m reference value. A maximum difference of 10%, as compared to the reference solution, is deemed acceptable for a simplified method. This difference decreases while increasing the PDF bound such that ~~RBFS~~ AFS is found to have the same range of applicability as that of MC with a maximum difference of the order of 10%. The criterion "AFS ≥ 1" gave also 94 m and 86 m for bounded PDF and k_R equal to 1.39 and 2, respectively. The range of applicability of the criterion "AFS ≥ 1" is indicated in terms of D/C ratios in Figure 13. If we accept a difference of 10% with the reference solution, the use of "AFS ≥ 1" is restricted to k_R value smaller than 2.5. On the other hand, comparing AFS to FS_{req} allows to introduce an additional safety margin. This added safety margin obviously yields to significantly lower allowable H_w and smaller D/C ratios as compared to other safety formats.

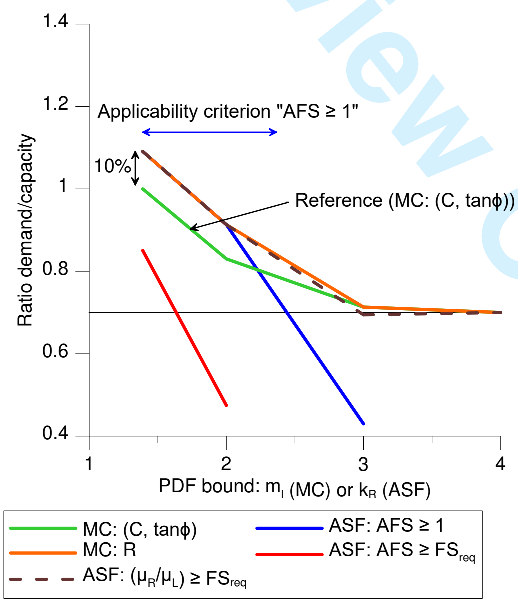

Figure 13. Demand/Capacity ratios for ~~RBSF~~ AFS as compared to MC, for varying PDF bounds in MC simulations (m_l) and ~~RBSF~~ AFS (k_R)

From a practical standpoint, we feel confident to allow H_w equal to 83 m (3 m of overtopping) for the ICOLD Benchmark dam, using the "AFS ≥ FS_{req}" ~~RBSF~~ criterion with $k_R = 1.39$ corresponding to the 5% fractile of the shear strength parameters. After performing the "sophisticated" analyses presented in this paper, we checked our findings from McCann et al. (1985) against rules of thumb for the allowable depth of overtopping, h_o, as a function of the dam height, using h = 80 m (262.4 ft), in our case. Preliminary screening values for h_o are based on field experience and engineering judgement. They were recommended using the following description and equations (in feet) (i) for dams in good condition: with very little seepage, no cracks or movement ($h_o = h^{0.6} \approx 8.6$ m), (ii) for dams in a fair condition: with moderate seepage, small structural cracks, slight differential movement ($h_o = h^{0.45} - 1 \approx 3.5$m) and (iii) for dams in poor conditions: with excessive seepage, large continuous cracks, excessive differential movements ($h_o = h^{0.3} - 1 \approx 1$ m). The proposed ~~RBSF~~ "AFS ≥ FS_{req}" criterion yielded a h_o value of 3 m corresponding to the rule of thumb for a dam in fair condition. This sounds about right considering the potential scour at the downstream toe, the vibrations induced by the overflowing aerated water nappe, the increased in downstream toe uplift pressure due to the downstream face water jet changing direction at the toe. Such phenomenon are not being considered explicitly in dam safety guidelines, and are very difficult to model from a sound probabilistic standpoint.

6. Conclusions

In this paper, the consideration of material uncertainties in gravity dam sliding stability assessment was investigated for four safety evaluation formats: (i) deterministic, (ii) semi-probabilistic (partial coefficient), (iii) reliability-based Adjustable Factor of Safety (AFS), and (iv) probabilistic Monte Carlo (MC) analysis. The results were presented in terms of the allowable water level, H_w, demand/capacity ratios (D/C), and FS to reach an unstable condition. In AFS and MC, the selected target failure probability, p_f*, was 10^{-5}. A 80 m-high gravity dam was used for applications without considering uncertainties in the applied loads, L. The main conclusions can be summarised as follows:

- The dam engineering profession shows a huge interest in comprehensive probabilistic methods. However, computation of p_f is very sensitive to the selection of shear strength random variables and PDF data as shown by the wide dispersion observed from the ICOLD Benchmark's results for a 80 m-high gravity dam. Practical applications are thus challenging, and generalisation of probabilistic analyses needs clear guidance.
- Using the deterministic format, H_w was found to be 103 m as compared to a reference MC solution with H_w equal to 90 m. The Deterministic safety format was found inadequate to introduce uncertainties even with an arbitrary reduction of the required FS if material tests are conducted
- In MC analyses, PDF bound data are the predominant parameters affecting the computation of p_f. A simplified bounded MC solution, using a single force resultant shear strength random variable, R, yielded a dam capacity approximately 10% larger than a bounded reference MC solution using two random variables (cohesion, C and friction, tanϕ). This excessive capacity decreases as the PDF bound becomes larger, the difference becoming insignificant when unbounded distributions are considered.
- A reliability-based adjustable safety factor (~~RBFS~~ AFS) is a simplified and practical approach to introduce probabilistic uncertainties in shear strength parameters. The

criterion "(μ_R/μ_L) $\geq FS_{req}$", using direct integration to compute FS_{req}, yielded the same results as MC simulations using the same PDF data and a single random variable, R. This criterion is recommended as a preliminary substitute to full probabilistic MC analysis that may therefore be avoided within the scope of the gravity dam stability problem studied herein. The proposed simplified approach yields a good accuracy with a 10% maximum difference with respect to a more comprehensive MC reference solution. The criterion "AFS $\geq FS_{req}$", which introduces an additional safety margin, cannot be compared the MC reference solution, because no additional safety margin was introduced in the MC solution. "AFS $\geq FS_{req}$" yielded a lower allowable water level (83 m) than the MC solution (90 m). However, $H_w = 83$ m does correspond to an established rule of thumb to estimate the capacity of a 80 m-high gravity dam.

- The proposed ~~RBSF~~ AFS method is a practical answer to the need for a simplified and robust method to introduce material data shear strength uncertainties using a rigorous approach in gravity dam stability analysis. Its range of applicability and the adequacy of the safety margin provided as compared to reference MC solutions make ~~RBSF~~ AFS a useful tool to use in sensitivity analysis of PDF data before undertaking more comprehensive MC analyses (or variants such as FORM).

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Appendix 1 – ICOLD Benchmark material data

The fifteen couples of friction angle ϕ (°) and cohesion C (kPa) specified as input "material test" data in the 11th ICOLD Numerical Benchmark (Escuder-Bueno, Altarejos-Garcia and Serrano-Lombillo 2011) are: (ϕ, C)={(45, 500); (37, 300); (46, 300); (45, 700); (49, 800); (53, 200); (54, 600); (45, 0); (49, 100); (60, 200); (63, 200); (62, 400); (60, 700); (56, 100); (62, 400)}.